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Tangent bundle of a Riemannian manifold

Definition. Horizontal sub-bundle of double tangent bundle of a Riemannian manifold

Let (M, g) be a Riemannian manifold and

$$\begin{aligned} \pi_{TM} : TM &\rightarrow M \\ X_p &\mapsto p \end{aligned}$$

be the projection from its tangent bundle.

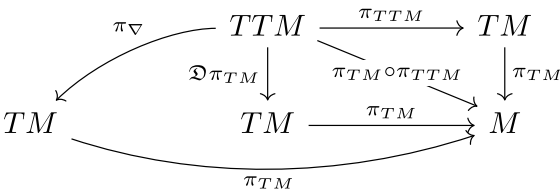
Consider the **connector map**

$$\begin{aligned} \pi_{\nabla} : TTM &\rightarrow TM \\ \beta'(0) &\mapsto \nabla_{\pi_{TM*}\beta'(0)}\beta \end{aligned}$$

whose kernel

$$\ker(\pi_{\nabla}) \leq TTM$$

is the **horizontal sub-bundle**.



- We have the product of projection maps

$$D\pi_{TM} \times \pi_{\nabla} : TTM \rightarrow TM \times TM$$

- We have the split

$$TTM = \underbrace{\ker(D\pi_{TM})}_{\text{vertical } V} + \underbrace{\ker(\pi_{\nabla})}_{\text{horizontal } H}$$

Sasaki metric

Definition. Saski metric on tangent bundle of a Riemannian manifold

$$\begin{aligned} T_u TM \times T_u TM &\rightarrow \mathbb{R} \\ (V + H) \times (V + H) &\rightarrow \mathbb{R} \\ g_u^{\text{Sas}}(v_1 + w_1, v_2 + w_2) &:= g_{\pi(u)}(v_1, v_2) + g_{\pi(u)}(w_1, w_2) \end{aligned}$$

Proposition: Let (M, g) be a Riemannian manifold. Then g -geodesics in TM are g^{Sas} -geodesics.

exact symplectic form

By the diffeomorphism

$$\begin{aligned} g : TM &\rightarrow T^*M \\ X_p &\mapsto g_p(X_p, -) \end{aligned}$$

we have the 1-form

$$\begin{aligned} T_u TM &\rightarrow \mathbb{R} \\ (V + H) &\rightarrow \mathbb{R} \\ \alpha_u(v + w) &:= g_{\pi(u)}(u, v) \end{aligned}$$

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