Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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Fibonacci generating function

$$
\begin{aligned}
f(x) :=& 1x + 1x^2 + 2x^3 + \dots \\
xf(x) =& \qquad 1x^2 + 1x^3 + \dots \\
(1-x)f(x) =& 1x + 0x^2 + \underbrace{1x^3 + \dots}_{x^2 f(x)} \\
(1-x)f(x) =& x + \ x^2 f(x)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
f(x) &= \frac{x}{1-x-x^2} \\
&= \frac{-x}{\left(x+\frac{1+\sqrt{5}}{2}\right)\left(x-\frac{1+\sqrt{5}}{2}\right)} \\
&= \frac{1}{\sqrt{5}}\left(\frac{\frac{1-\sqrt{5}}{2}}{x+\frac{1-\sqrt{5}}{2}} - \frac{\frac{1+\sqrt{5}}{2}}{x+\frac{1+\sqrt{5}}{2}}\right) \\
&= \frac{1}{\sqrt{5}}\left(\frac{1}{1-\left(\frac{1+\sqrt{5}}{2}\right)x} - \frac{1}{1-\left(\frac{1-\sqrt{5}}{2}\right)x}\right)
\end{aligned}
$$

This is expected because $\frac{1}{1-ax} = \sum a^n x^n$ makes

$$
\begin{aligned}
&\frac{1}{\sqrt{5}}\left(\frac{1}{1-\left(\frac{1+\sqrt{5}}{2}\right)x} - \frac{1}{1-\left(\frac{1-\sqrt{5}}{2}\right)x}\right) \\
=& \sum_{n\geq 0} \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)x^n
\end{aligned}
$$

where

$$
\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)
$$

is the n-th Fibonacci number.

Current note has 0 direct children and 0 total descendants.

- stretchy
  - seq
    - Fibonacci sequence
      - Fibonacci generating function

And it has 1 siblings.

- stretchy
  - seq
    - Fibonacci sequence
      - Fibonacci generating function