ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on December 10, 2025 1:57:40 AM,
was last modified on December 10, 2025 3:26:27 AM, and
was exported on September 7, 2026 3:24:49 PM.

Stalk of a pre-sheaf

pre-sheaf of Set

Definition. Stalk of pre-sheaf of Set

Let $\mathscr{S}$ be a pre-sheaf of Set on a topological space X. Then the **stalk** $\mathscr{S}_x$ at a point $x \in X$ is

$$\mathscr{S}_x := \frac{\{(U,s) | s \in \mathscr{S}(U), x \in U \text{ (open)} \subseteq X\}}{(U,s) \sim (V,t) \iff \exists W \subset U \cap W : \mathrm{res}_W s = \mathrm{res}_W t}$$

For any U (open) $\subseteq X$ and $s \in \mathscr{S}(U)$ we denote the equivalence class of (U,s) in $\mathscr{S}_x$ as $[s]_x$.

etale space

Definition. Etale space of a pre-sheaf of Set

Let $\mathscr{S}$ be a pre-sheaf of Set on a topological space X. Consider the set

$$|\mathscr{S}| := \coprod_{x \in X} \mathscr{S}_x$$

with the projection

$$\pi_{|\mathscr{S}|} : |\mathscr{S}| \to X$$

Given $s \in \mathscr{S}(U)$, we have a **section**

$$\begin{aligned} [s] : U &\to |\mathscr{S}| \\ x &\mapsto [s]_x \end{aligned}$$

and consider the topology generated by image of open sets under sections

$$\mathfrak{T}_{|\mathscr{S}|} := \langle \{[s](U) | U \text{ (open)} \subseteq X, s \in \mathscr{S}(U)\} \rangle$$

The **etale space** of $\mathscr{S}$ is the set $|\mathscr{S}|$ over X, equipped with the topology $\mathfrak{T}_{|\mathscr{S}|}$.

> **Proposition:**
>
> - $$\pi_{|\mathscr{S}|} : |\mathscr{S}| \to X$$
>   is continuous and is a local homeomorphism?.
> - The sections $[s]$ are continuous.

Let $U \subseteq X$ be open. Then

$$\pi^{-1}_{|\mathscr{S}|}(U) = \bigcup_{x \in U} \mathscr{S}_x = \bigcup_{V \subset U} \bigcup_{s \in \mathscr{S}(U)} [s](V) \in \mathfrak{T}_{|\mathscr{S}|}$$

- ...

Definition. Sheaf associated to a pre-sheaf of Set

Let $\mathscr{S}$ be a pre-sheaf of Set on a topological space X and

$$\pi_{|\mathscr{S}|} : |\mathscr{S}| \to X$$

the projection from its etale space. Then the sheaf of sections of π

$$\tilde{\mathscr{S}}(U) := \left\{\alpha : U \to |\mathscr{S}| \text{ continuous } \middle| \pi_{|\mathscr{S}|} \circ \alpha = \mathsf{Id}_U \right\}$$

is the sheaf associated to $\mathscr{S}$.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Sheaf on open sets of a topological space
      - Stalk of a pre-sheaf

And it has 1 siblings.

- structures based on sets
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