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Iterative dynamics of a continuous function on a topological space

Definition. Iterative dynamics of a continuous function on a topological space

Let

$$f : X \rightarrow X$$

be a continuous function on a topological space X . This defines a dynamical system in the usual sense

$$\begin{aligned} X \times \mathbb{N} &\rightarrow X \\ (x, i) &\mapsto f^i(x) \end{aligned}$$

Definition. Definition

forward orbit	$\text{orb}_f^+(x) := \{f^i(x) \mid i \in \mathbb{Z}_{\geq 0}\}$
backward orbit	
omega-limit set	$\omega_f(x) := \bigcap_{n \in \mathbb{Z}_{\geq 0}} \overline{\{f^k(x) : k \geq n\}}$
alpha-limit set	
invariant sets	$f(A) \subseteq A$
strongly invariant	$f(A) = A$
CINE sets	closed, invariant and non-empty sets
minimal sets	CINE set but does not properly contain any other CINE set

[1]

Definition. Dynamics that are topologically transitive and mixing

f is called	if
topologically transitive (or irreducible)	for every pair of open sets U, V , there is an iteration of U which does not intersect V $\exists k \in \mathbb{N} : f^k(U) \cap V = \emptyset$
topologically exact (locally eventually onto)	for every open set U , there is an integer k such that $f^k(U) = X$
minimal	every orbit is dense, that is, for all $x \in X$ $\overline{\{f^k(x) \mid k \in \mathbb{Z}\}} = X$
topologically mixing	for any two non-empty open sets U, V , the iterations of U eventually stays intersected with V , that is, $\exists N \in \mathbb{N} : n > N \implies f^n(U) \cap V \neq \emptyset$

sensitive dependence on initial conditions	with a metric Definition. Sensitive dependence on initial conditions of dynamics of a map between metric space Let (X, d) be a metric space and $f : X \rightarrow X$ be a map. If there exists a $\epsilon > 0$ such that $\forall x \in X, \delta > 0 \left(\exists y \in X, n \in \mathbb{Z}_{>0} \left(d(f^n(x), f^n(y)) \geq \epsilon \right) \right)$then dynamics of f is said to exhibit ϵ-sensitive dependence on initial conditions.
chaotic map	(with a metric) it is topologically transitive, dense set of periodic orbits and has <i>sensitive dependence on initial conditions</i>

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1. etheses.bham.ac.uk/id/eprint/1476/1/Barwell_11_PhD.pdf ↩