Info

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is written (completely with human hands) by Rupadarshi Ray,
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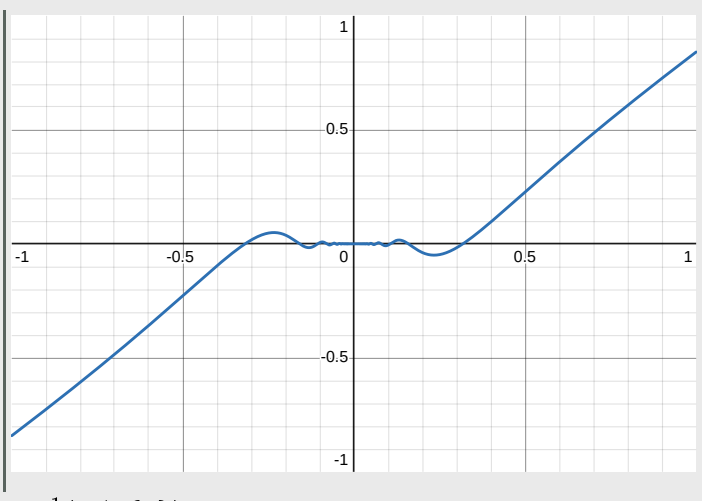
$$x + x^2 \sin \frac{1}{x}$$

From

Proposition:

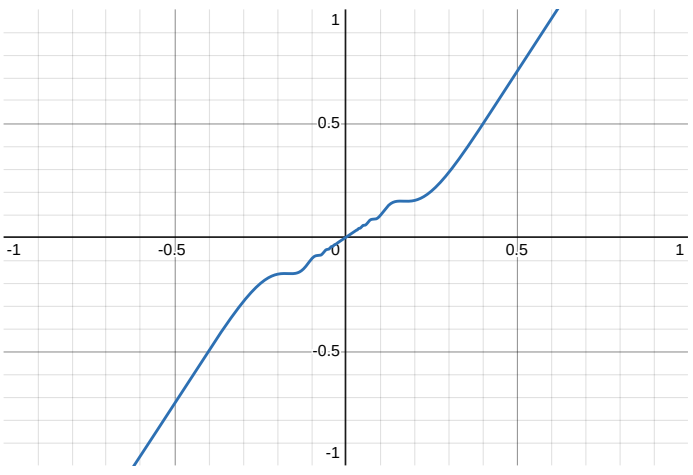
$$x^2 \sin \frac{1}{x}$$



is $\mathcal{C}^1(\mathbb{R} \setminus \{0\})$ and differentiable at 0 but derivative is not continuous at 0.

we know

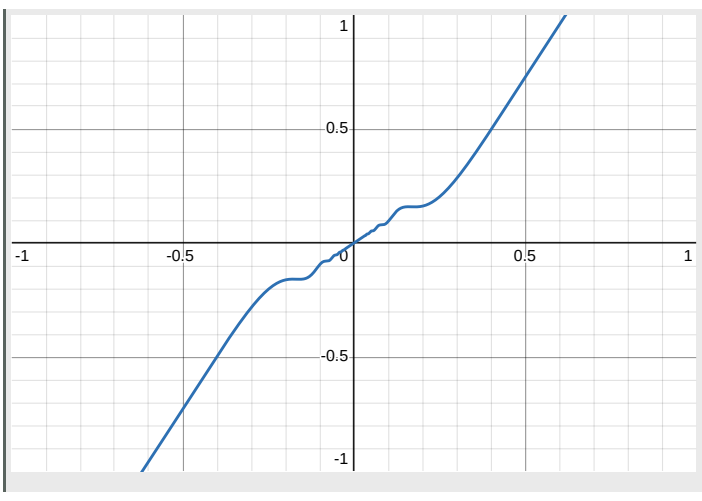
$$\frac{\mathrm{d}}{\mathrm{d}x} x + x^2 \sin \frac{1}{x}\bigg|_{x=0} = 1$$



However,

Proposition: The differentiable map

$$\begin{aligned} \mathbb{R} &\to \mathbb{R} \\ x &\mapsto x + x^2 \sin \frac{1}{x} \end{aligned}$$



is $\mathcal{C}^\infty(\mathbb{R} \setminus \{0\})$ and has derivative 1 at 0, however it not injective on any onb of 0.

The derivative is continuous on $\mathbb{R} \setminus \{0\}$ and changes sign on every onb of 0. Thus by

> Let $f : I \text{ (open interval)} \subseteq \mathbb{R} \to \mathbb{R}$ be differentiable. Then f is strictly monotonic
> $\iff$
> $Z(f')$ does not contain an open interval/has empty interior/every stationary point is isolated and
> f' does not change sign on $I \setminus Z(f')$.

we conclude the function is not injective/monotonic.

[1]

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $x + x^2 \sin \frac{1}{x}$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $\cos \sqrt{z}$
    - The complex logarithm
    - $\sum_{n \geq 0} z^{n!}$

- $z^3 - 3z$
- $$\frac{r_0}{1 + \epsilon \cos b\theta}$$
- $\cos\left(\pi \frac{p}{q}\right)$
- $\frac{\cos x^k}{x}$
- Elliptic integrals and Jacobi elliptic functions
- The complex exponential $\exp(z)$
- $\exp\left(\frac{z+1}{z-1}\right)$
- e^{-x^2}
- [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.]]$
- $\int \frac{1}{x^2+bx+c}$
- $\frac{\exp(z)}{1-z}$
- $\sin x$
- $\sin(x \sin(x))$
- $\frac{\sin x}{x}$
- stock.sin x by xk
- $\sin x^2$
- $\sqrt{z}$
- $\sqrt{z^2 - 1}$
- Theta functions
- Weierstrass elliptic function $\wp$
- Weierstrass' primary factors
- $x^2 \sin \frac{1}{x}$
- $x + x^2 \sin \frac{1}{x}$

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1. https://math.stackexchange.com/a/2356140/1290493 ↩