Info

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Extending holomorphic functions by reflections

> **Proposition:** Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under $\mathbb{C}$-conjugation
> $$\Omega^\dagger = \Omega$$
> and
> $$f : \Omega \to \mathbb{C}$$
> be continuous and holomorphic on $\Omega \setminus \mathbb{R}$. Then f is holomorphic on Ω.

- By

  > **(Morera's theorem)** Any continuous function $f : U \to \mathbb{C}$ on open set $U \subseteq \mathbb{C}$ that satisfies
  > $$\int_\gamma f(z)\mathrm{d}z = 0$$
  > for every $\mathcal{C}^1$ closed curve γ in U must be holomorphic on U.

  for triangles.

> **(Schwarz reflection principle)** Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under conjugation
> $$\Omega^\dagger = \Omega$$
> and
> $$f : \Omega \cap H \to \mathbb{C}$$
> be a holomorphic function that extends continuously to
> $$\Omega \cap \mathbb{R} \to \mathbb{R}$$
> Then f extends onto Ω by
> $$f(z) = \overline{f(\bar{z})} \text{ for } z \in \bar{\Omega}$$
> (uniquely by identity).

- $$f(\bar{z}) = \sum_{n \geq 0} a_n (\bar{z} - \bar{z}_0)^n$$

  then

  $$\overline{f(\bar{z})} = \sum_{n \geq 0} \bar{a}_n (z - z_0)^n$$

  is holomorphic on Ω^-

  - But for $x \in \Omega \cap \mathbb{R}$ we have $\overline{f(x)} = f(x)$ so f extends continuously on Ω.
  - Now by

    > **Proposition:** Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under $\mathbb{C}$-conjugation
    > $$\Omega^\dagger = \Omega$$
    > and
    > $$f : \Omega \to \mathbb{C}$$
    > be continuous and holomorphic on $\Omega \setminus \mathbb{R}$. Then f is holomorphic on Ω.

    it is holomorphic.

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Extending holomorphic functions by reflections

And it has 19 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Approximation of holomorphic functions on $\mathbb{C}$ by rational functions
      - Embedding unit disk D into $\mathbb{C}^3$
      - Factorization of holomorphic functions on $\mathbb{C}$
      - Global holomorphic functions
      - Equivalent descriptions of holomorphic functions
      - Meromorphic functions on the complex plane
      - Holomorphic functions meromorphic at infinity
      - Modular forms
      - Reconstructing meromorphic functions from stalks
      - Extending holomorphic functions by reflections
      - Rotational symmetrization of holomorphic functions
      - Sheaf of holomorphic functions on $\mathbb{C}$

- $\mathscr{O}(U)$
- $\mathscr{O}(\mathbb{C})$
- $\mathscr{O}(D)$
- $\mathscr{O}^p(H^2_{\mathrm{U}})$
- $\mathscr{O} \cap L^p$
- $\mathscr{O}(S^1)$
- Zeros and singularities of holomorphic functions