

Info

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The sheaf of differential forms on open subsets of manifolds

Definition. $\Omega^k, Z\Omega^k$ sheaves on $\text{Open}(M)$

$$\Omega^k, Z\Omega^k : \text{Open}(M)^{op} \rightarrow \text{Vec}_{\mathbb{R}}$$

$$\Omega^k(U) := \Gamma_U \Lambda^k T^*(M)$$
$$Z\Omega^k(U) := \ker d^{k+1}|_{\Gamma_U \Lambda^k T^*(M)}$$

- the restriction is the usual restriction of differential forms

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are sheaves

Exact k -forms $d\Omega^{k-1}$ do not form a sheaf!

d^k morphism on sheaves

$$0 \rightarrow \mathbb{R} \rightarrow \cdots \rightarrow \Omega^k \xrightarrow{d^k} \Omega^{k+1}$$

is an exact sequence of sheaves.

By Poincare lemma!

Isomorphism of the de Rham cohomology with the sheaf cohomology of locally $\mathbb{R}$ -constant functions

$$H^p(M, \mathbb{R}) \cong H^p(\Omega^\bullet(M))$$

double complex combinatorics on a countable? good cover

On a countable cover, a general MV principle gives

$$H^k(\text{Forms}^\bullet) \cong H^k_{\text{double}}(C^\bullet(\mathfrak{U}, \text{Forms}^\bullet))$$

And on a good cover, we have the right side

$$H^k(\text{Forms}^\bullet(M)) \cong H^k_{\text{double}}(C^\bullet(\mathfrak{U}, \text{Forms}^\bullet)) \cong H^k(C^\bullet(\mathfrak{U}, \mathbb{R}))$$

direct limit of a constant

$$\begin{aligned} \check{H}^k(M, \mathbb{R}) &:= \lim_{\text{good } \mathfrak{U}} \underbrace{H^k(C^\bullet(\mathfrak{U}, \mathbb{R}))}_{\cong H^k(\text{Forms}^\bullet(M))} \\ &\cong H^k(\text{Forms}^\bullet(M)) \end{aligned}$$

Current note has 1 direct children and 1 total descendants.

- [structures based on sets](#)
 - [manifolds](#)
 - [smooth \$\mathbb{R}\$ -manifold](#)
 - [Tangents on a smooth manifold](#)
 - [Differential forms on smooth manifolds](#)
 - [The sheaf of differential forms on open subsets of manifolds](#)
 - [Isomorphism of the de Rham cohomology with the sheaf cohomology of locally \$\mathbb{R}\$ -constant functions](#)

And it has 7 siblings.

- [structures based on sets](#)
 - [manifolds](#)
 - [smooth \$\mathbb{R}\$ -manifold](#)
 - [Tangents on a smooth manifold](#)
 - [Differential forms on smooth manifolds](#)
 - [Differential 1-forms on smooth manifolds](#)
 - [Chain complex of differential forms on a smooth manifold](#)
 - [Exterior derivative operator on space of smooth differential forms](#)
 - [Functoriality of \$\text{Forms}^k\$ on smooth manifolds](#)

- The integral map on space of differential forms
- Three different relative chain complexes of differential forms
- The sheaf of differential forms on open subsets of manifolds