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Fractional Sobolev spaces

Definition. Definition

For any $s \in \mathbb{R}$ the function

$$\xi \mapsto (1 + |\xi|^2)^{s/2} \in \mathcal{C}^\infty(\mathbb{R}^d)$$

and is *slowly increasing*.

The map

$$\begin{aligned} \Lambda_s : \mathcal{S}^\star &\rightarrow \mathcal{S}^\star \\ f &\mapsto ((1 + |\xi|^2)^{s/2} \hat{f})^\vee \end{aligned}$$

is a linear homeomorphism with inverse Λ_{-s} .

Definition. Definition

$$H^s := \{f \in \mathcal{S}^\star \mid \Lambda_s f \in L^2\}$$

[1]

Current note has 0 direct children and 0 total descendants.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Differentiable functions
 - Distributional derivatives
 - Fractional Sobolev spaces

And it has 4 siblings.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Differentiable functions
 - Distributional derivatives
 - Fractional Sobolev spaces
 - $H^1(U)$
 - H^1_{loc}
 - $H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$

1. [Gerald B Folland - Real Analysis Modern Techniques and Their Applications 2nd edition \(1999\),_p.318](#)

