Info

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S^1

#wiki/Man/R/1

Definition. S^1

The smooth compact 1-manifold **circle** is

$$S^1 := \{(x,y) \in \mathbb{R}^2 \big| x^2 + y^2 = 1\}$$

Double derivative/Laplace operator on the circle

- cat
- t - types/extension
- m - making/structures/more structure
    - $$\mathsf{S}^1 := \frac{[0,1]}{0 \sim 1}$$
    - standard topology
    - standard differential structure
    - standard Riemannian metric
    - Lie group structure
- p - constructing more/attachments/constructed with
    - products Torus $(\mathsf{S}^1)^n$
        - 2-torus
    - sum
        - wedge sum of circles

- v - points/insides/properties
    - $$\pi_1(\mathsf{S}^1,(1,0)) \cong_{\mathsf{Grp}} (\mathbb{Z},+)$$
- s - subsets
- f - functions/equations(polynomials, equations, differential equations)/dynamics
    - Homeomorphisms $S^1 \to S^1$
- b - function spaces, attachments, bundles and duals
    - $$T\mathsf{S}^1 \cong_{\boldsymbol{VecBun}\mathsf{Top}} \mathsf{S}^1 \times \mathbb{R}$$
    - $\boldsymbol{VecBun}_k\mathsf{Top}(\mathsf{S}^1)$
        - cylinder
            - $S^1 \times [0,1]$
            - $S^1 \times \mathbb{R}$
        - mobius bundle
          $$\frac{[0,1] \times \mathbb{R}}{(0,t) \sim (1,-t)}$$
    - Upto isomorphism, every real vector bundle over the circle is either trivial, or the Whitney sum of a trivial bundle with the Mobius bundle
    - $\mathrm{L}^p(\mathsf{S}^1,\mu)$ space.R.sphere.1.L 2
        - S^1-module
        - Fourier : $\mathrm{L}^2(S^1,\mu) \to \mathrm{L}^2(S^1,\mu)$

- o - external effects
- e - equivalences
    - automorphisms
        - $$\mathrm{Diff}(\mathsf{S}^1)$$
        - $$\mathrm{AutRiem}(\mathsf{S}^1) \cong O_{\mathbb{R}}(2)?$$
        - $$\mathrm{AutLieGrp}(\mathsf{S}^1)$$
    - identifications
        - $\mathsf{S}^1 \subseteq \mathbb{R}^2 = \mathbb{C}\mathsf{S}^1 \subseteq \mathbb{C}$
        - $$\mathbb{R}/\mathbb{Z} = \mathsf{S}^1$$
        - $\mathsf{S}^1 \cong (\mathbb{C}^{\mathrm{unit}}, \times) \cong_{\mathsf{LieGrp}} U_{\mathbb{C}}(1) \cong SO_{\mathbb{R}}(2)$

- Knots in $\mathbb{R}^3$

Current note has 4 direct children and 4 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - $\mathscr{O}(S^1)$
    - An
      - Analytic functions on the circle
- stretchy
  - S^1
    - The map $z \mapsto z^n$ on the circle
    - Left translation $S^1 \curvearrowright S^1$
    - Homeomorphisms $S^1 \to S^1$
    - Loops in S^1 upto homotopy

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - $\overline{B^n}$
  - $\mathrm{BS}(1,2)$
  - $(\mathbb{C}, +, \times)$
  - $D^* := D \setminus \{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
  - H^3_2
  - $[a,b]$
  - Figure 8 knot
  - Lamplighter group on $\mathbb{Z}_2$
  - $\beta\mathbb{N}$
  - $\#_2\mathbb{R}P^2$
  - $\prod_{i\in\mathbb{N}}\{1,\ldots,n_i\}$
  - $\mathbb{R}P^n$
  - $(\mathbb{Q}, +, \times, <)$
  - $\mathbb{Q}_{\hat{p}}$
  - $\overline{\mathbb{Q}}$
  - $\mathbb{Q}_{\text{constr}}$
  - $\mathbb{Q}(\xi_n)$
  - $\mathcal{O}_{\overline{\mathbb{Q}}}$
  - $(\mathbb{R}, +, \times, <, \sup)$
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset
  - $\mathbb{R}^2 \times S^1$
  - $\mathbb{R}^n$
  - $\mathbb{R} \times S^1$
  - $\mathbb{R} \times S^2$
  - S^1
  - $S^1 \wedge S^1$
  - $S^1 \times S^n$
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
  - Mapping torus of the antipodal map of S^2
  - S^3
  - S^n
  - I-bundle
  - Symmetric group
  - $\mathbb{T}$
  - T^2
  - T^2_g
  - $T^2_{0,0,3}$
  - $\mathbb{R}^2 \setminus \{0\}$
  - Three $T^2_{1,0,1}$ glued at their boundaries
  - $T^2_{1,1,0}$
  - T^2_2
  - $T^2_g \times [0,1]$
  - $\#_{\mathbb{N}}T^2$
  - Thompson's group
  - T^n
  - 3-valent tree
  - 4-valent tree
  - $[0,1]^{\mathbb{N}}$ with product topology
  - Whitehead manifolds
  - $\mathbb{Z}$
  - $\mathbb{Z}[\sqrt{m}]$
  - $\mathbb{Z}[X]$
  - $\mathbb{Z}^{\mathbb{N}}$
  - $\mathbb{Z}[i]$

> - $\mathbb{Z}[i\sqrt{5}]$