Info

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Fibers of holomorphic mappings

zeros of holomorphic functions are isolated

- If $f : U \to \mathbb{C}$ is a non-constant holomorphic function with

$$f(z) \equiv \sum_{n \geq k} a_n (z - z_0)^n \qquad \text{on } B(r, z_0)$$

for $z_0 \in U$ where k is the smallest positive integer such that $a_k \neq 0$.
  - $\implies$

$$f(z) = (z - z_0)^k g(z) \qquad \text{on } B(r_1, z_0) \subseteq B(r, z_0)$$

    where g is a holomorphic function such that

$$g(z) :\equiv a_k + a_{k+1}(z - z_0) + \dots$$

  - As $g(z_0) = a_k \neq 0$, this means there is a smaller disk on which g is non-zero

$$g \neq 0 \qquad \text{on } B(r_2, z_0)$$

  - $\implies$ $f \neq 0$ on that same punchered disk around z_0
  - Thus zeroes of non-constant holomorphic functions on an open subset of $\mathbb{C}$ are isolated in $\mathbb{C}$, that is, the set of zeros $Z(f)$ is discrete in $\mathbb{C}$.

fibers of holomorphic functions are isolated

(Identity theorem for holomorphic functions on subsets of $\mathbb{C}$) Let

$$f : \Omega(\text{open}) \subseteq \mathbb{C} \to \mathbb{C}$$

be holomorphic. And L be the set of limit points of $f^{-1}(0)$. Then L **is both open and closed in** Ω.
Therefore if Ω is connected, either $L = \Omega \iff f \equiv 0 \text{ on } \Omega$ or $f^{-1}(0)$ has no limit point in Ω, that is if $f^{-1}(0)$ has a limit point in Ω then $f \equiv 0$.

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And it has 13 siblings.