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Fibers of holomorphic mappings

zeros of holomorphic functions are isolated

- If $f : U \rightarrow \mathbb{C}$ is a non-constant holomorphic function with

$$f(z) \equiv \sum_{n \geq k} a_n (z - z_0)^n \qquad \text{on } B(r, z_0)$$

for $z_0 \in U$ where k is the smallest positive integer such that $a_k \neq 0$.

- $\implies$

$$f(z) = (z - z_0)^k g(z) \qquad \text{on } B(r_1, z_0) \subseteq B(r, z_0)$$

where g is a holomorphic function such that

$$g(z) :\equiv a_k + a_{k+1}(z - z_0) + \dots$$

- As $g(z_0) = a_k \neq 0$, this means there is a smaller disk on which g is non-zero

$$g \neq 0 \qquad \text{on } B(r_2, z_0)$$

- $\implies f \neq 0$ on that same punched disk around z_0
- Thus zeroes of non-constant holomorphic functions on an open subset of $\mathbb{C}$ are isolated in $\mathbb{C}$, that is, the set of zeros $Z(f)$ is discrete in $\mathbb{C}$.

fibers of holomorphic functions are isolated

(Identity theorem for holomorphic functions on subsets of $\mathbb{C}$) Let

$$f : \Omega(\text{open}) \subseteq \mathbb{C} \rightarrow \mathbb{C}$$

be holomorphic. And L be the set of limit points of $f^{-1}(0)$. Then **L is both open and closed in Ω** .

Therefore if Ω is connected, either $L = \Omega \iff f \equiv 0$ on Ω or $f^{-1}(0)$ has no limit point in Ω , that is if $f^{-1}(0)$ has a limit point in Ω then $f \equiv 0$.

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- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [mapping](#)
 - [Fibers of holomorphic mappings](#)

And it has 13 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [mapping](#)
 - [Bounding derivative of holomorphic mappings](#)
 - [Bounding holomorphic functions linearly](#)
 - [Branch points and ramification index of holomorphic functions](#)
 - [Disk in holomorphic image of disk](#)
 - [Size of holomorphic image of disks](#)
 - [Maximum and minimum of holomorphic functions](#)
 - [Fibers of holomorphic mappings](#)
 - [Non-constant holomorphic functions locally look like \$w^k\$ and are open mappings](#)
 - [Biholomorphic mapping onto disk](#)
 - [Conformal radius of pointed simply connected open subsets of \$\mathbb{C}\$](#)
 - [Holomorphic functions on punched disk](#)
 - [Holomorphic function whose singularities are not isolated](#)
 - [A holomorphic map branched over \$\mathbb{Q}\$](#)