

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on September 6, 2026 9:53:56 PM, was last modified on September 6, 2026 10:16:24 PM, and was exported on September 7, 2026 3:24:55 PM.

Balls in dual of a topological vector space

weak[★] compactness of the closed unit ball

 **(Banach-Alaoglu)** Let X be a normed vector space. The closed unit ball

$$\overline{B(0,1)} := \{f \in X^\star \mid \|f\|_{X \rightarrow \mathbb{C}} \leq 1\}$$

is compact in the weak[★] topology.

 By Tychonoff's theorem

$$\prod_{x \in X} \overline{B(0, \|x\|_X)}$$

is a compact space and consider the embedding

$$\Phi := \prod_{x \in X} \text{ev}_x : \overline{B(0,1)} \rightarrow \prod_{x \in X} \overline{B(0, \|x\|_X)} \\ l \mapsto (l(x))_{x \in X}$$

|
• ...

[1]

Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Topological vector spaces over \$\mathbb{R}\$ or \$\mathbb{C}\$](#)
 - [Balls in dual of a topological vector space](#)

And it has 7 siblings.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Topological vector spaces over \$\mathbb{R}\$ or \$\mathbb{C}\$](#)
 - [Banach spaces](#)
 - [Dual space of a topological vector space](#)
 - [Balls in dual of a topological vector space](#)
 - [Convex compact subsets of weak[★] dual](#)
 - [Weak[★] topology on dual of a topological vector space](#)
 - [Compact endomorphisms](#)
 - [Weak topology on a topological vector space](#)

1. [Einsiedler and Ward - Functional Analysis, Spectral Theory, and Applications, p.256](#) ↩