Lebesgue measure of boundary of open sets in $\mathbb{R}^n$

for any $\alpha, \beta > 0$ there exists an open set of measure $\leq \alpha$ whose boundary has measure $\geq \beta$

- Let $\alpha, \beta > 0$ and K be closure of an open set such that $m(K) = \alpha + \beta$.
  - Let
    $$\{s_m\}_{m \in \mathbb{N}} \subseteq K$$
    be a countable dense sequence.
  - Consider a sequence $\{\alpha(m)\}_{m \in \mathbb{N}} \subseteq (0, \infty)$ such that
    $$m(B_1) \sum_{n \in \mathbb{N}} \alpha(n)^n = \alpha$$
    for example
    $$\alpha(n)^n :\equiv \frac{\alpha}{m(B_1)} \frac{1}{2^{n-1}}$$
  - Then we have
    $$U := \bigcup_{m \in \mathbb{N}} B_{\alpha(m)}(s_m) \cap K \subseteq K$$
  - So U is an open set with $\overline{U} = K$ and
    $$m(U) \leq m(B_1) \sum_{n \in \mathbb{N}} \alpha(m)^n = \alpha$$
    and
    $$\begin{aligned} \operatorname{int}(U) \sqcup \partial U &= \overline{U} \\ U \sqcup \partial U &= K \\ \underbrace{m(U)}_{\leq \alpha} + m(\partial U) &= \underbrace{m(K)}_{\alpha + \beta} \end{aligned}$$
  - Therefore, for arbitrary $\alpha, \beta > 0$ we have an open set U of measure α whose boundary ∂U has measure $\geq \beta$.

[1]

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measure of boundary of open sets in $\mathbb{R}^n$

And it has 39 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary
    - Complex ODEs in $\mathbb{C}$
    - Circle packing on $\mathbb{R}^2$
    - Circle packing converges to the Riemann biholomorphism
    - Bounded connected open subsets of $\mathbb{C}^n$
    - Connected circular open subsets of $\mathbb{C}^n$
    - Continuous functions on $\mathbb{R}^d$
    - Dyadic cubes
    - Curves
    - Differentiable functions
    - Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner transform
    - Harmonic functions composed with conformal maps
    - Hilbert transform
    - Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Holomorphic subsets of $\mathbb{C}^n$
    - Regular hypersurfaces in $\mathbb{R}^{2n}$
    - Orientable hypersurfaces in $\mathbb{R}^n$
    - Laplace operator on $\mathbb{R}^n$
    - $l^\infty(\mathbb{R})$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - Local $\mathbb{R}$-algebras
    - $l^p(\mathbb{R})$
    - Metric density of subsets of $\mathbb{R}^n$
    - Monotone functions on $\mathbb{R}$
    - Cauchy integral of periodic functions
    - Polygons with integer vertices

- Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$

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1. analysis - Does the boundary of an open set have measure zero (in $\mathbb{R}^n$)? - Mathematics Stack Exchange ↩