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Hermitian manifolds

Definition. Hermitian metric on a almost complex manifold

A **Hermitian metric** on a almost complex manifold (M, J) is a $\mathcal{C}^\infty(M, \mathbb{R})$ -bilinear map

$$h : \Gamma(TM) \times \Gamma(TM) \rightarrow \mathcal{C}^\infty(M, \mathbb{C})$$

such that

- on (TM, J) it is $\mathcal{C}^\infty(M, \mathbb{C})$ -sesquilinear

$$h(JX, Y) = ih(X, Y)$$

- conjugate symmetric

$$h(X, Y) = \overline{h(Y, X)}$$

- positive

$$X \neq 0 \implies h(X, X) > 0$$

Definition. Hermitian manifolds

Let (M, J) be a complex manifold with a Hermitian metric

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Then (M, J, h) is called a **Hermitian manifold** where

- $g := \Re h$ is the **fundamental Riemannian metric** of h
- $\omega := -\Im h$ is the **fundamental 2-form** of h

- $$h = g - i\omega$$

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 - [Hermitian manifolds](#)

And it has 20 siblings.

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