Info

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Subsets of topological groups

Definition. Subsets of topological groups

Let G be a topological group and $A, B \subset G$ and $x \in G$.

$$\begin{aligned} Ax &:= \{yx | y \in A\} = R_x(A) \\ xA &:= \{xy | y \in A\} = L_x(A) \\ A^{-1} &:= \{y^{-1} | y \in A\} \\ AB &:= \{xy | x \in A, y \in B\} = \bigcup_{x \in A} xB = \bigcup_{y \in B} Ay \\ A^n &:= \{x^n | x \in A\} \\ A^{(n)} &:= \{x_1 \dots x_n | x_1, \dots, x_n \in A\} \end{aligned}$$

- $$\underbrace{A^2}_{\{x^2 | x \in A\}} \subset \underbrace{AA}_{\{xy | x, y \in A\}}$$

For $A \subseteq G$ $\xrightarrow{\text{if}}$ $\downarrow$ then	A is open	A is closed	A is compact
Ax	open	closed	compact
xA	open	closed	compact
A^{-1}	open	closed	compact
AB	open	?	-
$A^n := \{x^n \| x \in A\}$	?	?	?
$A^{(n)} = A(A^{(n-1)})$	open		compact
$\bigcup_{n>0} A^{(n)}$	open		-

Let $V \subseteq G$ be open

- **Base case:**

$$V^{(2)} = \{gx | g, x \in V\} = \bigcup_{g \in V} \underbrace{gV}_{\text{open}}$$

is open.

- **Induction hypothesis:** For $n = k - 1$

$$V^{(n)}$$

is open.

- **Induction step:** We see

$$V^{(k)} = \bigcup_{g \in V} \underbrace{gV^{(k-1)}}_{\text{open}}$$

is open.

Thus $V^{(k)}$ is open for all $k > 0$.

symmetric sets

Definition. Let G be a topological group. We say $A \subset G$ is **symmetric** if $A = A^{-1}$.

> **Proposition:**
>
> $$A \cap B = \emptyset \iff i_G \notin A^{-1}B$$

Let G be a topological group. For every onb U of i_G there is a symmetric onb V of i_G such that

$$VV \subseteq U$$

Continuity of

$$(x, y) \mapsto xy$$

at

$$(i_G, i_G) \mapsto i_G$$

means for every onb U of i_G there are onb W_1, W_2 of i_G with $W_1 W_2 \subseteq U$.

- Consider the set

$$V := W_1 \cap W_2 \cap W_1^{-1} \cap W_2^{-1}$$

- $$V^{-1} = V, VV \subseteq W_1 W_2 \subseteq U$$

Let G be a topological group. Any subgroup $H \leq G$ and its closure $\overline{H}$ is a topological group with the subspace topology.

Every open subgroup of a topological group G is a closed subgroup.

Let S be a open subgroup of G. Then it is precisely the complement of all the non-trivial cosets

$$S = G \setminus \underbrace{\bigcup_{g \notin S} \underbrace{gS}_{\text{open}}}_{\text{closed}}$$

Hence, S is closed.

Proposition: If $A, B \subset G$ are compact sets then AB are also compact.

$$\underbrace{A \times B}_{\text{compact}} \mapsto \underbrace{AB}_{\text{image of compact}}$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Subsets of topological groups

And it has 7 siblings.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Uniformly continuous functions on a topological group
      - Any onb of identity of a connected topological group generates it
      - Image of discrete subgroups of topological groups
      - Rigidity of discrete groups in topological groups
      - Subsets of topological groups
      - Locally compact Hausdorff topological group
      - Translation on a topological group