> Info
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
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Uniformly continuous functions on a topological group

> **Definition.** Uniformly continuous functions on a topological group
>
> A function $f : G \to \mathbb{C}$ is **left uniformly continuous** if
>
> $$\|\tau_y f - f\|_\infty \xrightarrow{y \to i_G} 0$$
>
> and **right uniformly continuous** if
>
> $$\left\|R_y^* f - f\right\|_\infty \xrightarrow{y \to i_G} 0$$

> Let G be a topological group and $f \in \mathcal{C}_c(G)$. Then f is left and right uniformly continuous
>
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Given $f \in \mathcal{C}_c(G)$ and $\epsilon > 0$. Let $K := \operatorname{supp} f$.

- Let $x \in K$, by continuity of f we have

  $$f^{-1}B_{\epsilon/2}(f(x))$$

  is open and contains x. Then

  $$U_x := x^{-1}f^{-1}B_{\epsilon/2}(f(x))$$

  is open and contains i_G.
- Thus, for every $x \in K$ there is a neighborhood U_x of i_G such that

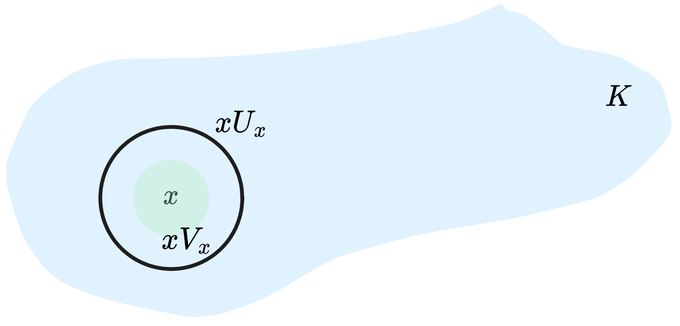
  $$\begin{aligned} & y \in U_x \\ \iff & xy \in xU_x \implies |f(xy) - f(x)| < \frac{\epsilon}{2} \end{aligned}$$

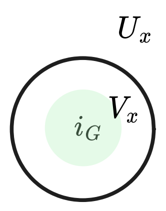
  and by

  > Let G be a topological group. For every onb U of i_G there is a symmetric onb V of i_G such that
  >
  > $$VV \subseteq U$$
  >
  > Continuity of
  >
  > $$(x, y) \mapsto xy$$
  >
  > at
  >
  > $$(i_G, i_G) \mapsto i_G$$
  >
  > means for every onb U of i_G there are onb W_1, W_2 of i_G with $W_1W_2 \subseteq U$.
  >
  > - Consider the set
  >
  > $$V := W_1 \cap W_2 \cap W_1^{-1} \cap W_2^{-1}$$
  >
  > - $$V^{-1} = V, VV \subseteq W_1W_2 \subseteq U$$

  there is a symmetric neighborhood V_x of i_G such that

  $$V_xV_x \subset U_x$$

- Now

  $$K = \bigcup_{x \in K} \{xV_x | x \in K\}$$

but by compactness of K we have

$$K \subset \bigcup_{i=1}^{n} x_i V_{x_i}$$

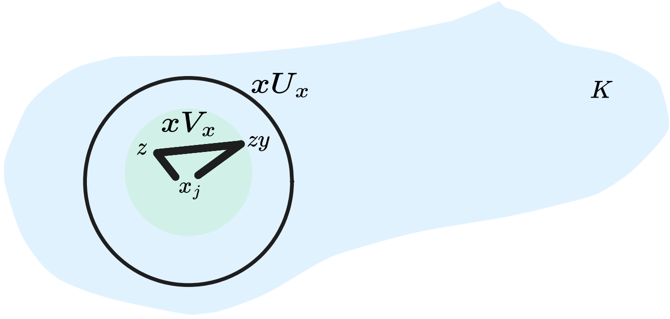
- Let

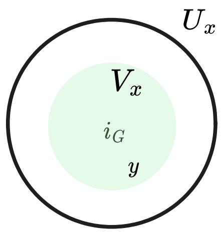
$$V := \bigcap_{j=1}^{n} V_{x_j}$$

which is a neighborhood of i_G

- > **Proposition:**
  >
  > $$y \in V \implies \left\| R_y^* f - f \right\|_\infty < \epsilon$$
  >
  > that is
  >
  > $$\forall z \in G, y \in V \implies |f(zy) - f(z)| < \epsilon$$

  - $$\begin{aligned} z \in K &\implies \exists x_j : z &&\in x_j V_{x_j} \\ &\implies x_j^{-1} z &&\in V_{x_j} \\ y \in V &\implies y &&\in V_{x_j} \\ &\implies x_j^{-1} zy &&\in V_{x_j} V_{x_j} \\ & &&\subset U_{x_j} \\ &\implies zy &&= x_j x_j^{-1} zy \\ & &&\in x_j U_{x_j} \end{aligned}$$





then

$$\begin{aligned} |f(zy) - f(z)| &\le |f(zy) - f(x_j)| + |f(x_j) - f(z)| \\ &\le \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &\le \epsilon \end{aligned}$$

  - if $z \notin K$ then $f(z) = 0$
    - if $zy \notin K$, then

    $$f(x) = 0 = f(xy)$$

    so

    $$|f(xy) - f(x)| = 0 < \epsilon$$

    - if $zy \in K$ then

    $$\begin{aligned} &\exists x_j : x_j^{-1} zy \in V_{x_j} \\ &\implies x_j^{-1} z \in U_{x_j} \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Uniformly continuous functions on a topological group

And it has 7 siblings.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Uniformly continuous functions on a topological group
      - Any onb of identity of a connected topological group generates it
      - Image of discrete subgroups of topological groups
      - Rigidity of discrete groups in topological groups
      - Subsets of topological groups
      - Locally compact Hausdorff topological group
      - Translation on a topological group