Info

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Any onb of identity of a connected topological group generates it

Let G be a topological group. For every onb U of i_G there is a symmetric onb V of i_G such that

$$VV \subseteq U$$

Continuity of

$$(x, y) \mapsto xy$$

at

$$(i_G, i_G) \mapsto i_G$$

means for every onb U of i_G there are onb W_1, W_2 of i_G with $W_1 W_2 \subseteq U$.

- Consider the set

$$V := W_1 \cap W_2 \cap W_1^{-1} \cap W_2^{-1}$$

- $$V^{-1} = V, VV \subseteq W_1 W_2 \subseteq U$$

Let G be a connected topological group and

$$V \subseteq G$$

be an open neighbourhood of i_G with $V = V^{-1}$.

We have

$$V^{(k)} := \{g_1 \dots g_k | g_1, \dots, g_k \in V\}$$

Then the group generated by V is

$$S = \bigcup_{k>0} V^{(k)}$$

By

For $A \subseteq G$ $\xrightarrow{\text{if}}$ $\downarrow\text{then}$	**A is open**	**A is closed**	**A is compact**
Ax	open	closed	compact
xA	open	closed	compact
A^{-1}	open	closed	compact
AB	open	?	-
$A^n := \{x^n \| x \in A\}$	?	?	?
$A^{(n)} = A(A^{(n-1)})$	open		compact
$\bigcup_{n>0} A^{(n)}$	open		-

we know

$$V^{(k)}$$

is open

Thus

$$S = \underbrace{\bigcup_{k>0} \underbrace{V^{(k)}}_{\text{open}}}_{\text{open}}$$

is an open subgroup.

Every open subgroup of a topological group G is a closed subgroup.

Let S be a open subgroup of G. Then it is precisely the complement of all the non-trivial cosets

$$S = G \setminus \underbrace{\bigcup_{g \notin S} \underbrace{gS}_{\text{open}}}_{\text{closed}}$$

Hence, S is closed.

Thus S is both open and closed subset of G which is connected, implying

$$G = \bigcup_{k>0} V^{(k)}$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Any onb of identity of a connected topological group generates it

And it has 7 siblings.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Uniformly continuous functions on a topological group
      - Any onb of identity of a connected topological group generates it
      - Image of discrete subgroups of topological groups
      - Rigidity of discrete groups in topological groups
      - Subsets of topological groups
      - Locally compact Hausdorff topological group
      - Translation on a topological group