ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on May 21, 2026 10:47:32 PM,
was last modified on May 21, 2026 11:59:14 PM, and
was exported on September 7, 2026 2:50:32 PM.

# Angle function

Consider the function

$$\theta : \mathbb{R}^2 \setminus (-\infty, 0] \to \mathbb{R}$$

such that

$$\mathrm{d}\theta = \frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2 + y^2}$$

and

$$\theta(1, 0) = 0$$

Then

$$\lim_{y \to 0+} \theta(x, y) = \pi$$
$$\lim_{y \to 0-} \theta(x, y) = -\pi$$

## bounded derivative but non-Lipshitz

The operator norm of the derivative

$$\|\mathrm{d}\theta\| = \frac{\sqrt{x^2 + y^2}}{x^2 + y^2} = \frac{1}{\sqrt{x^2 + y^2}}$$

is bounded on complement of the unit disk

$$\|\mathrm{d}\theta\| \leq 1 \text{ on } \left(\mathbb{R}^2 \setminus (-\infty, 0)\right) \setminus \overline{D^2}$$

However,

$$\frac{|\theta(-2, \epsilon) - \theta(-2, -\epsilon)|}{|(-2, \epsilon) - (-2, -\epsilon)|} \xrightarrow{\epsilon \to 0} \frac{2\pi}{2\epsilon}$$

Therefore,

$$|\theta(y) - \theta(x)| \leq \underbrace{\|\mathrm{d}\theta\|}_{\leq 1} |y - x|$$

does not hold.

[1]

---

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - Angle function

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos \sqrt{z}$$
    - The complex logarithm
    - $\sum_{n \geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1 + \epsilon \cos b\theta}$$
    - $\cos\left(\pi \frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - $e^{-x^2}$
    - [Only sum terms that are multiples of $N$ in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$
      ]]
    - $\int \frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x \sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2 - 1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$

- Weierstrass' primary factors
- $x^2 \sin \frac{1}{x}$
- $x + x^2 \sin \frac{1}{x}$

---

1. https://math.stackexchange.com/questions/704108/a-generalization-of-the-mean-value-theorem ↩