Info

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Counting compact topological manifolds upto homeomorphism

[1]

without boundary

Suppose there are not countably many of them, then there are an uncountable number of them for some dimension say n. Let

$$\{M_\alpha \mid \alpha \in A\}$$

be such a collection.

For each α we find a collection of imbeddings for $1 \leq j \leq k_\alpha$

$$h_{\alpha j} : B(2) \subset \mathbb{R}^n \to M_\alpha$$

such that

$$\{h_{\alpha j}(B(1)) | 1 \leq j \leq k_\alpha\} \text{ covers } M_\alpha$$

We may assume

- $k_\alpha = k$ for all α by choosing a uncountable subcollection
- $h_{\alpha j} \mid_{B(1)}$ can be extended to an imbedding of $B(k+1) \to M_\alpha$
- $M_\alpha \subset \mathbb{R}^{2n+1}$

$$\epsilon_{\alpha jm} := d(h_{\alpha j}(B(m)), M_\alpha - h_{\alpha j}(B(m+1)))$$
$$\epsilon_\alpha := \min_{j,m}\{\epsilon_{\alpha jm}\}$$

We again assume $\exists \epsilon > 0$ such that $\epsilon_\alpha > \epsilon$ for all $\alpha \in A$ by choosing a subcollection.

Each M_α determines an imbedding

$$g_\alpha : B(k+1) \to (\mathbb{R}^{2n+1})^k$$
$$g_\alpha(x) := (h_{\alpha,1}(x), \ldots, h_{\alpha,k}(x))$$

The set of all such imbeddings $\{g_\alpha \mid \alpha \in A\}$ under the uniform metric

$$d_u(g_\alpha, g_\beta) := \max_{x \in B(k+1)} d(g_\alpha(x), g_\beta(x))$$

is a separable metric.

Hence, g_{α_0} is a limit point of a sequence of distinct imbeddings $g_{x_1}, \ldots$.

> **Proposition:** $M_{\alpha_0} \cong M_{\alpha_i}$ for i sufficiently large. Furthermore, this homeomorphism can be taken close to $\iota : M_{\alpha_0} \hookrightarrow \mathbb{R}^n$ arbitrarily d-close.

Definition. Let

$$V_j(m) := h_{\alpha_0 j}(B(m))$$

for $1 \leq j \leq k, 1 \leq m \leq k+1$ and

$$V_j'(m) := h_{\alpha_i j}(B(m)) \subset M'$$

for $M' = M_{\alpha_i}$ for large enough i.

Definition. Now let

$$U_j(m) := \bigcup_{p=1}^{j} V_p(m)$$

Note $U_k(1) = M$ and $U_k'(1) = M$.

Definition.

$$f_j : V_j(k+1) \to V_j'(k+1)$$
$$h_{\alpha_i, j} \circ h_{\alpha_0, j}^{-1}$$

Suppose we construct an imbedding

$$g_j : U_j(m) \to M'$$

ϵ-close to identity by choosing M' far enough in the sequence.

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- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold topological
      - Counting compact topological manifolds upto homeomorphism

And it has 2 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold topological

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1. Counting topological manifolds - ScienceDirect ↩