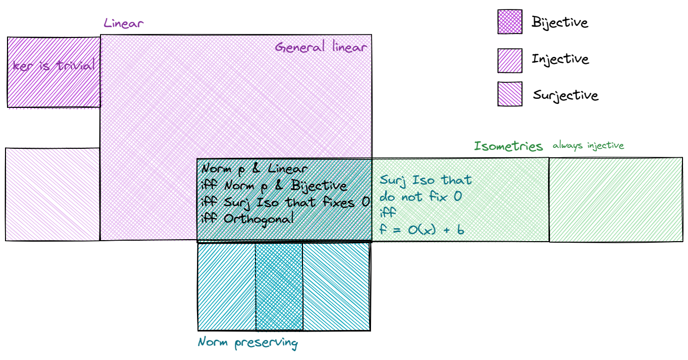
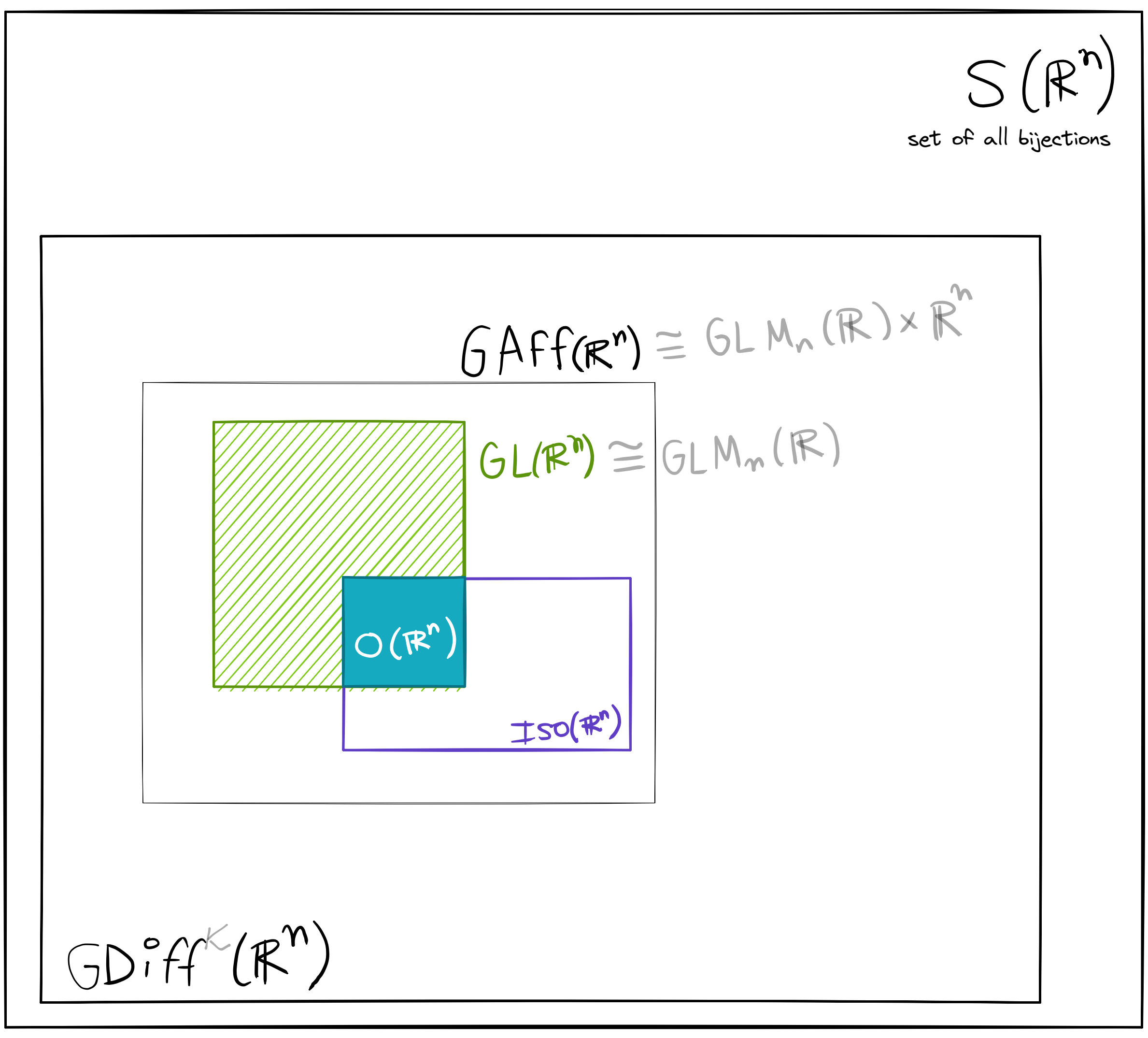
Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on September 20, 2022 1:51:02 PM,
was last modified on September 7, 2026 2:32:06 PM, and
was exported on September 7, 2026 3:04:07 PM.

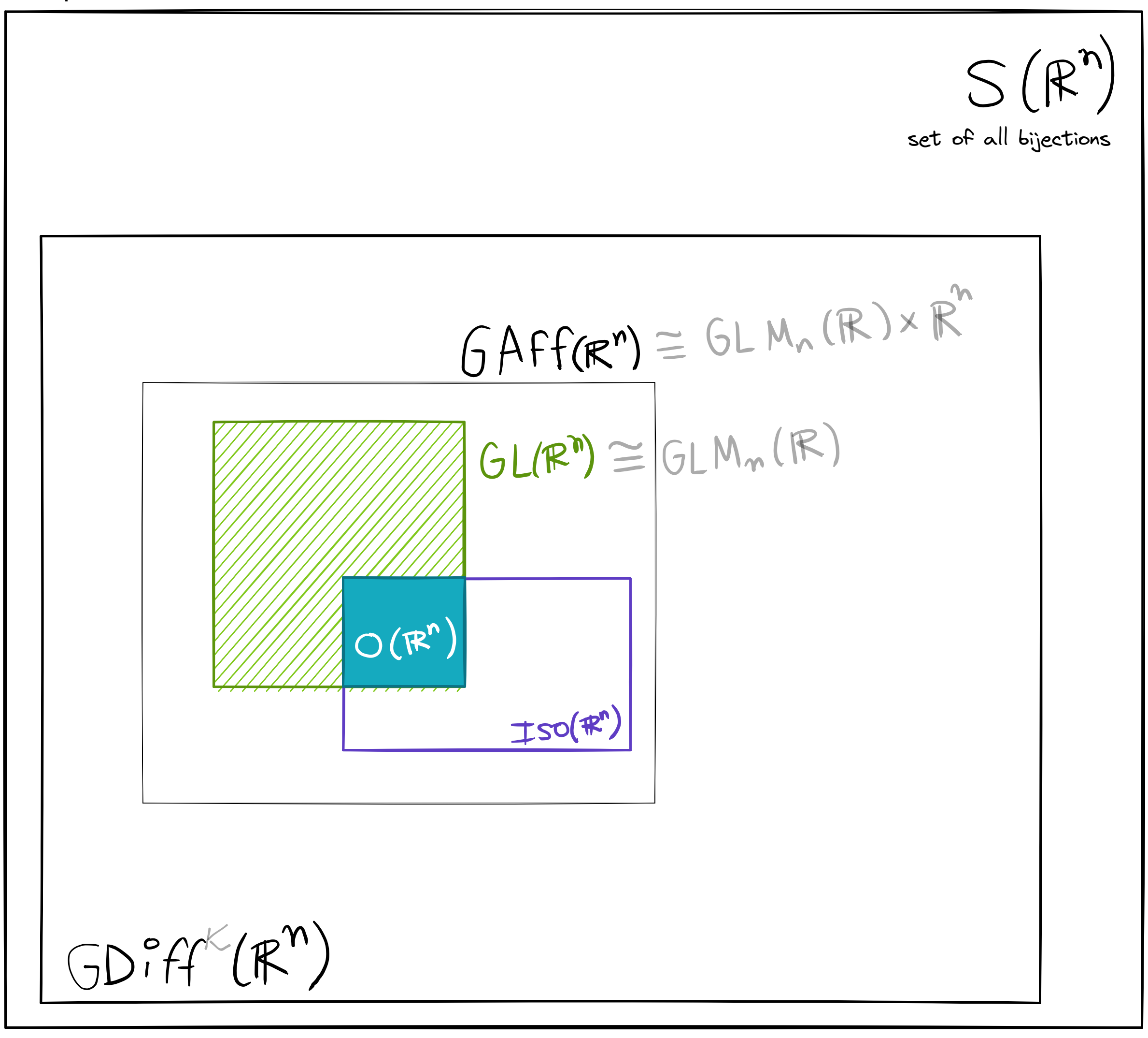
# Automorphisms of $\mathbb{R}^n$




⚠ Invertible Transformations of $\mathbb{R}^n$




- Map




finition]

$$(S(\mathbb{R}^n), \circ)$$

be the symmetric group of $\mathbb{R}^n$, the set of all bijections $f : \mathbb{R}^n \to \mathbb{R}^n$ with operation of composition.

This set is not closed under $+$ (function addition) because $+-$identity is not a bijection.

## translations group

Definition. $\mathrm{Transl}(\mathbb{R}^n)$

For some $\mathbf{x} \in \mathbb{R}^n$, $\mathcal{T}_{\mathbf{x}} : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$\mathcal{T}\mathbf{x}(\mathbf{v}) := \mathbf{v} + \mathbf{x}$$

$$(\mathrm{Transl}(\mathbb{R}^n), \circ) \leq (S(\mathbb{R}^n), \circ)$$

$(\mathrm{Transl}(\mathbb{R}^n), \circ, *)$ is a $n$-dimensional $\mathbb{R}$-vector space where

$$\alpha * \mathcal{T}_{\mathbf{x}} := \mathcal{T}_{\alpha\mathbf{x}}$$

- space.R.n.Aut.GL

## Isometries

Definition. $\mathrm{Iso}(\mathbb{R}^n)$

$f \in S(\mathbb{R}^n)$ such that

$$\|\mathbf{x} - \mathbf{y}\| = \|f(\mathbf{x}) - f(\mathbf{y})\|$$

then $f \in \mathrm{Iso}(\mathbb{R}^n)$ and is called an **isometry**.

### subgroups of isometries

$$(\mathrm{Transl}(\mathbb{R}^n), \circ) \leq (\mathrm{Iso}(\mathbb{R}^n), \circ)$$

$$(O(\mathbb{R}^n), \circ) \leq (\mathrm{Iso}(\mathbb{R}^n), \circ)$$

### finding all isometries!

$f \in \mathrm{Iso}(\mathbb{R}^n)$ such that $f(\mathbf{0}) = \mathbf{0}$ (fixes origin), then $f \in O(\mathbb{R}^n)$.

For any $f \in \mathrm{Iso}(\mathbb{R}^n)$, $\exists O_f \in O(\mathbb{R}^n)$, $\mathbf{x}_0 \in \mathbb{R}^n$ such that

$$f(\mathbf{x}) = O_f(\mathbf{x}) + \mathbf{x}_0$$

## Affine transformations

Definition. $GA(\mathbb{R}^n)$

## Symmetries!

Definition. $\mathrm{Symm}(S)$

**Symmetry** of a subset $S \subseteq \mathbb{R}^n$ is the set $\mathrm{Symm}(S)$ defined by
$f \in \mathrm{Iso}(\mathbb{R}^n)$ such that

$$f(S) \subseteq S$$

then $f \in \mathrm{Symm}(S)$.

Trivially,

$S \subseteq f(S)$

## diffeomorphism group

Definition. $\mathrm{Diff}^{\,r}(\mathbb{R}^n)$

$f \in S(\mathbb{R}^n)$ such that $f \in \mathcal{C}^r(\mathbb{R}^n, \mathbb{R}^n)$ and $f^{-1} \in \mathcal{C}^r(\mathbb{R}^n, \mathbb{R}^n)$ then $f \in \mathrm{Diff}^{\,r}(\mathbb{R}^n)$ and is called a $r$-**diffeomorphism** for any $r \in \mathbb{N} \cup \{\infty\}$.

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Current note has 1 direct children and 1 total descendants.

- stretchy
  - $\mathbb{R}^n$
    - Automorphisms of $\mathbb{R}^n$
      - $\mathrm{AutPoly}(\mathbb{R}^n)$

And it has 4 siblings.

- stretchy
  - $\mathbb{R}^n$
    - Automorphisms of $\mathbb{R}^n$
    - $GL(n, \mathbb{R}^n) \curvearrowright \mathbb{R}^n$
    - Isometric embedding of a subset of $\mathbb{R}^n$ into $\mathbb{R}^n$
    - Dot product 2-norm on $\mathbb{R}^n$