Info

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$U(\mathbb{C}\setminus\{0\})$

#wiki/Man/R/2

We construct the connected, simply connected, 2 dimensional smooth $\mathbb{R}$-manifold (diffeomorphic to $\mathbb{R}^2$) that is the universal cover of $\mathbb{R}^2\setminus\{(0,0)\}$.

Definition.

$$\begin{tikzcd} \mathbb{Z} \arrow[r, phantom, "\curvearrowright"] & (U(\mathbb{C}^\times),+) \arrow[d, "\pi"] \\ & (\mathbb{C}^\times,\times) \end{tikzcd}$$

Consider

$$\mathbb{C}^\times := \mathbb{C}\setminus\{0\}$$

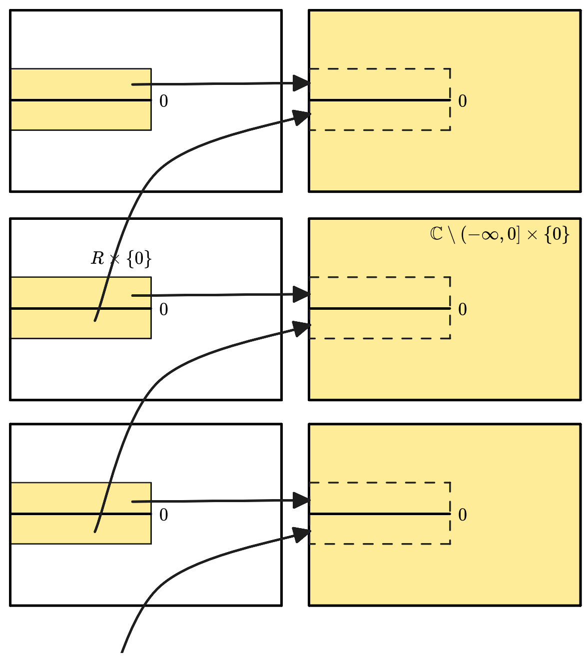
and take the open subset $\mathbb{C}\setminus(-\infty,0]$ and the open rectangle $R:=(-\infty,0)\times(-1,1)$ and take $\mathbb{Z}$ many copies

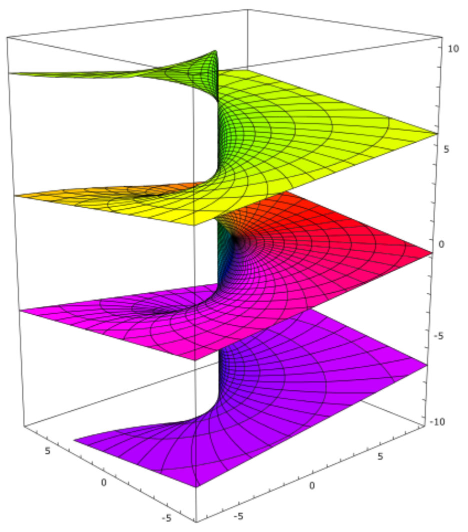
$$\begin{gathered} \mathbb{C}\setminus(-\infty,0]\times\mathbb{Z} \\ R\times\mathbb{Z} \end{gathered}$$

and consider the gluing functions

$$\begin{aligned} g_k : (R\setminus(-\infty,0))\times\{k\} &\to \mathbb{C}\setminus(-\infty,0]\times\mathbb{Z} \\ (z,k) &\mapsto \begin{cases} (z,k) & \Im z>0 \\ (z,k+1) & \Im z<0 \end{cases} \end{aligned}$$

for every $k\in\mathbb{Z}$





and glue them together using the gluing functions

$$\begin{aligned} U(\mathbb{C}^\times) := \frac{\mathbb{C}\setminus(-\infty,0]\times\mathbb{Z}\coprod R\times\mathbb{Z}}{g_k,\forall k\in\mathbb{Z}} &\xrightarrow{\pi} \mathbb{C}^\times \\ [z,k] &\mapsto z \end{aligned}$$

$U(\mathbb{C}^\times)$ is a Riemann surface and $(U(\mathbb{C}^\times),+)$ is a complex Lie group. The map

$$\begin{tikzcd} U(\mathbb{C}^\times) \arrow[d, "\pi"] \\ \mathbb{C}^\times \end{tikzcd}$$

is a holomorphic covering map and a group homomorphism.

embedding inside $\mathbb{R}^3$

$$\begin{aligned} U(\mathbb{R}^2\setminus\{0\}) &\hookrightarrow \mathbb{R}^3 \\ (x,y,\theta) &\mapsto (x,y,\theta) \end{aligned}$$

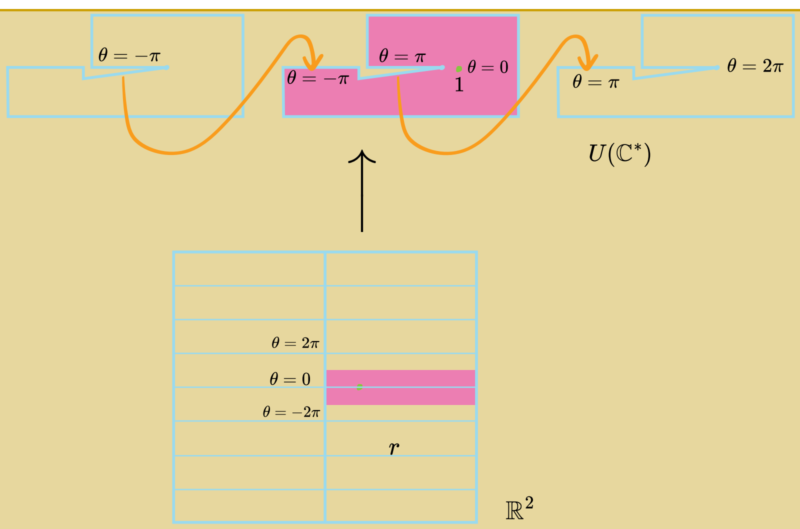
The image is a subset of

$$\mathbb{R}^2\setminus\{0\}\times\mathbb{R}$$

polar coordinates actually cover the universal cover!

The "polar coordinates on $\mathbb{R}^2\setminus\{0\}$" actually covers the infinite helicoid

$$\begin{aligned} \mathbb{R}_{>0}\times\mathbb{R} &\to U(\mathbb{C}^\times) \\ (r,\theta) &\mapsto (r\cos\theta, r\sin\theta,\theta) \end{aligned}$$



is a diffeomorphism.

which in complex coordinates this is

$$\begin{aligned} \mathbb{C}_{\mathrm{Re}>0} &\to U(\mathbb{C}^\times) \\ (r,\theta) &\mapsto (r\exp(i\theta),\theta) \end{aligned}$$

but it is of course **not** a bi-holomorphism?

A better way to think about this is: we think of

$$\begin{aligned} \mathbb{C}_{\mathrm{Re}>0} &\to \mathbb{C}^\times \\ (r,\theta) &\mapsto r\exp(i\theta) \end{aligned}$$

as the covering map π itself, where we think of the right half plane *as* the universal cover, (but still this is not holomorphic?).

$$\begin{tikzcd} \begin{array}{c}(0,\infty)\times\mathbb{R}\\ (|z|,\theta)\end{array} \arrow[rr, bend left] \arrow[dr] & & \begin{array}{c}U(\mathbb{C}\setminus\{0\})\\ (z,\theta)\end{array} \arrow[ll, bend left] \arrow[dl] \\ & \begin{array}{c}(0,\infty)\times S^1 \cong \mathbb{C}\setminus\{0\}\\ |z|e^{i\theta} = z\end{array} & \end{tikzcd}$$

We need

$$\exp : \mathbb{C} \to \mathbb{C}^\times$$

as the holomorphic map, which is the covering map.

the pullback of inexact closed forms on punchered plane

$$H^1(\Omega^\bullet(\mathbb{R}^2\setminus\{0\})) = \mathbb{R}\left[\frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2}\right]$$

$$\pi^*\frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2} = \mathrm{d}\theta$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - $U(\mathbb{C}\setminus\{0\})$

And it has 30 siblings.

- squishy
  - $SL(2)(\mathbb{R}) \curvearrowright \mathbb{R}^2$
  - $SL(2)(\mathbb{R}) \curvearrowright H^2_{\mathrm{U}}$
  - $\mathrm{Mat}_{\mathbb{C}}(2)$
  - $A_{r,-\langle z\mapsto\lambda z\rangle}\backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0\}_{,-\langle z\mapsto z+1\rangle}\backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0,1\}_{,-PSL(2)(\mathbb{Z})[2]}\backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{R}\ltimes_\theta\mathbb{C}^2$
  - $\boldsymbol{Cl}(\mathbb{R}^n,\langle\ ,\ \rangle_{0,n})$
  - $\mathbb{C}^n$
  - $U(\mathbb{C}\setminus\{0\})$
  - $PSL(2)(\mathbb{C}) \curvearrowright \mathbb{C}P^1 \cong_{\mathsf{Man}} S^2$
  - $\mathbb{C}P^n$
  - Farey graph, complex and tree
  - GL
  - Heisenberg group
  - Incomplete hyperbolic polygon mod side-pairing
  - $\mathbb{R}\boldsymbol{H}^2{}_{,-}\mathbb{R}H^2_{\mathrm{U}}{}_{,-}D^2$
  - $\mathbb{R}\boldsymbol{H}^n$
  - D_n lattice
  - Matrices
  - $\mathrm{Mat}_{\mathbb{R}}(2)$
  - Mobius endomorphisms
  - Elementary group with finite and infinite sided Dirichlet polyhedra
  - Polynomials
  - $T^*S^1 \not\cong_{\mathsf{Sym}} (\mathbb{R}^2\setminus\{0\}, \mathrm{d}x\wedge\mathrm{d}y)$
  - $Z\left(\sum_{1\le j\le n} z_j^2 - 1\right)(\mathbb{C}) \cong_{\mathsf{Sym}} T^*S^{n-1}$
  - SL
  - SO

- Seifert–Weber dodecahedral 3-manifold
- $U(n, \mathbb{C})$