> Info
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> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
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Decomposition of $f \in L^p$ into bounded and unbounded parts

Let $f \in L^1(\mathbb{R}^d)$ then

$$f = f\mathbb{1}_{\{|f| \le t\}} + f\mathbb{1}_{\{|f|>t\}}$$

Here, we have the *bounded part* of f

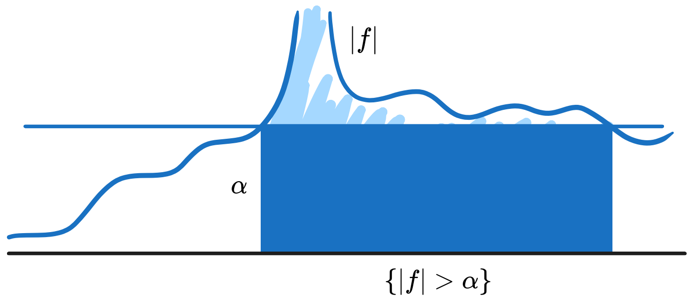
$$\begin{aligned} \left\| f\mathbb{1}_{\{|f|\le t\}} \right\|_1 &= \int_{\{|f|\le t\}} |f| \\ &\le \|f\|_1 \\ \text{and } \left\| f\mathbb{1}_{\{|f|\le t\}} \right\|_\infty &\le t \end{aligned}$$

and the *unbounded part*

$$\left\| f\mathbb{1}_{\{|f|>t\}} \right\|_1 = \int_{\{|f|>t\}} |f| \le \|f\|_1$$

and

$$m(\operatorname{supp}(f\mathbb{1}_{\{|f|>t\}})) = m(\{|f|>t\}) \le \frac{1}{t}\|f\|_1$$



> **(Chebyshev's inequality)** Let $f \in L^p(\mathbb{R}^d), p \in (0,\infty)$. Then for any $\alpha > 0$
>
> $$m\{|f|>\alpha\} \le \frac{\|f\|_p^p}{\alpha^p}$$
>
> $$\begin{aligned} \|f\|_p^p &= \int_{\mathbb{R}^d} |f|^p \\ &\ge \int_{\{|f|>\alpha\}} |f|^p \\ &\ge \alpha^p m\{|f|>\alpha\} \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Decomposition of $f \in L^p$ into bounded and unbounded parts

And it has 15 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Functions with uniformly bounded mean oscillations on cubes
      - Decomposition of $f \in L^p$ into bounded and unbounded parts
      - Calderon-Zygmund decomposition
      - Lebesgue density of measurable sets
      - Functions with uniformly bounded mean oscillations on dyadic cubes
      - Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0,1) \to \mathcal{C}[0,1)$
      - Measurable functions on $\mathbb{R}^n$
      - (ϵ, n)-measurable function
      - Integrability and integral of measurable functions on $\mathbb{R}^n$
      - Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
      - Lebesgue averaging and differentiation
      - Integrals of a monotonically converging sequence of functions
      - Lebesgue integral from measure of undergraph
      - $L^{p,q}$
      - $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$