> [!info] Info
>
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> is written (completely with human hands) by Rupadarshi Ray,
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Metric spaces

- types of subsets
  - balls
  - open
- properties of a subset
  - radius
  - open, closed
  - connected
  - compact
  - dense
  - nowhere dense
- *something* of a subset $2^M \to 2^M$
  - int, ext, boundary
  - adherent and accumulation points
  - closure of a subset
- properties of metric space
  - complete
  - separable
- metric
- curves
  - measuring length of curves
  - geodesics
  - geodesic completeness
  - length metric

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			int	**boundary**	**exterior**
			$\subseteq S$		$\subseteq S^c$
				boundary $=>$ limit or isolated	
closure $=$ lim pt $+$ isolated points			closure - interior $=$ boundary		
	limit point		✅	✅	**empty**
	isolated point			✅ isolated $\subseteq$ boundary	empty?
	non accumulation points			**empty**	✅

	----------	**int**	**boundary and in**	**boundary and out**
	lim point and in	✅	✅	**empty**
	isolated pt	**empty**	✅	**empty**
	lim point and outside	**empty**	**empty**	✅
	non-accu	**empty**	**empty**	**empty**

$$\text{limpt}() = \text{interior} \cup \underbrace{\partial\text{in} \cup \partial\text{out}}_{\partial \setminus \text{isolated}}$$

$$\text{closure}() = \text{interior} \cup \partial = () \cup \partial = \text{exterior}()^c$$

$$\partial() = \partial\text{in} \cup \partial\text{out} \cup \text{isolated}$$

1. interior $=>$ a limit point and in the set

$$\text{interior}()$$

2. isolated pt $=>$ *boundary*

$$\text{isolated}()$$

3. limit point in the set $=$ *boundary in the set*

$$\partial\text{in}()$$

4. limit point not in the set $=$ *boundary not in the set*

$$\partial\text{out}()$$

5. exterior $=$ not accumulation point

$$\text{exterior}()$$

$$() = \text{interior} \cup \underbrace{\text{isolated} \cup \partial\text{in}}_{\subseteq \partial}$$

$$\text{interior} \circ \text{interior}() = \text{interior}()$$

$$\text{interior}(A \cup B) = ?$$

$$\text{closure}(E^c) = \text{exteior}(E^c)^c = \text{interior}(E)^c$$

- 1
  - *closed*
    1. interior = limit point in the set
    2. isolated boundary
    3. limit point boundary in the set
    4. limit point boundary not in the set **empty**
    5. exterior = non-accumulation point
  - *empty exterior*
    1. interior = limit point in the set
    2. isolated boundary
    3. limit point boundary in the set
    4. limit point boundary not in the set
    5. exterior = non-accumulation point **empty**
  - *empty interior*
    1. interior = limit point in the set **empty**
    2. isolated boundary
    3. limit point boundary in the set
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
  - *no isolated boundary*
    1. interior = limit point in the set
    2. isolated boundary **empty**
    3. limit point boundary in the set
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
  - *open union isolated boundaries*
    1. interior = limit point in the set
    2. isolated boundary
    3. limit point boundary in the set **empty**
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
- 2
  - *open*
    1. interior = limit point in the set
    2. isolated boundary **empty**
    3. limit point boundary in the set **empty**
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
  - *all points are lim points*
    1. interior = limit point in the set
    2. isolated boundary **empty**
    3. limit point boundary in the set
    4. limit point boundary not in the set **empty**
    5. exterior = non-accumulation point
  - *all points are limit point boundaries* possible?
    1. interior = limit point in the set **empty**
    2. isolated boundary **empty**
    3. limit point boundary in the set
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
  - 1. interior = limit point in the set **empty**
    2. isolated boundary
    3. limit point boundary in the set **empty**
    4. limit point boundary not in the set
    5. exterior = non-accumulation point
  - 1. interior = limit point in the set **empty**
    2. isolated boundary
    3. limit point boundary in the set
    4. limit point boundary not in the set **empty**
    5. exterior = non-accumulation point
- 1,5 *no interior, no exterior* $\mathbb{Q} \subset \mathbb{R}$
  1. interior = limit point in the set**empty**
  2. isolated boundary
  3. limit point boundary in the set
  4. limit point boundary not in the set
  5. exterior = non-accumulation point**empty**
- 2,4 *no isolated boundary, empty exterior = dense*
  1. interior = limit point in the set
  2. isolated boundary**empty**
  3. limit point boundary in the set
  4. limit point boundary not in the set**empty**
  5. exterior = non-accumulation point
- 3,4 *only isolated boundaries* $\mathbb{N} \subset \mathbb{R}$
  1. interior = limit point in the set

2. isolated boundary
3. limit point boundary in the set **empty**
4. limit point boundary not in the set **empty**
5. exterior = non-accumulation point

- 3,5
  1. interior = limit point in the set
  2. isolated boundary
  3. limit point boundary in the set**empty**
  4. limit point boundary not in the set
  5. exterior = non-accumulation point **empty**
- 4,5 *closed and empty exterior => WHOLE SPACE*
  1. interior = limit point in the set
  2. isolated boundary
  3. limit point boundary in the set
  4. limit point boundary not in the set**empty**
  5. exterior = non-accumulation point**empty**

- 3
  - *perfect*
    1. interior = limit point in the set
    2. isolated boundary **empty**
    3. limit point boundary in the set **empty**
    4. limit point boundary not in the set **empty**
    5. exterior = non-accumulation point
  - 1. interior = limit point in the set
    2. isolated boundary
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    5. exterior = non-accumulation point
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  2. isolated boundary
  3. limit point boundary in the set
  4. limit point boundary not in the set
  5. exterior = non-accumulation point

Current note has 9 direct children and 19 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - adherent, accumulation points, closure of a subset in a metric space
      - Balls in a metric space
      - Totally bounded subsets of a metric space
      - Compact subsets of a metric space
        - Covering compactness in a metric space
        - Sequential compactness of a subset of a metric space
      - Covering of a subset of a metric space
      - Open cover of a compact metric space
      - Function between metric spaces
        - Continuous functions between metric spaces
        - Uniformly continuous functions on a metric space
        - Contraction map in a metric space
        - Topological entropy of maps between metric spaces
        - Limit of a function between metric spaces
        - Sensitive dependence on initial conditions of dynamics of a map between metric space

        - to
            - Functions from a metric space to $\mathbb{R}$
            - Sequence of functions from a metric space to $\mathbb{R}$
    - interior, exterior, boundary points in a metric space
    - open and closed sets in a metric space

And it has 11 siblings.

- structures based on sets
    - Metric spaces
        - Set of all metrics on a set
        - CAT(0)
        - Space of all compact metric spaces up to isometry
        - Y^X
        - Gromov hyperbolic spaces
        - Metric space isometries
        - Constructing more metrics
        - Metric spaces
        - Metric simplicial complexes
        - Metric subspaces
        - Uniformly discrete spaces of bounded geometry