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Sequences of holomorphic functions

uniform limit of holomorphic functions are holomorphic

We shall prove

Let $f_n : U \rightarrow \mathbb{C}$ be holomorphic on open $U \subseteq \mathbb{C}$ and

$$f_n \xrightarrow{\text{uniformly as } n \rightarrow \infty} f \text{ on } K$$

for every $K \subset U$ compact then f is holomorphic in U .

Choose a ball of some radius r around $z_0 \in U$

$$f_n(z) = \frac{1}{2\pi i} \int_{S^1_{r,z_0}} \frac{f_n(w)}{w - z} dz \xrightarrow{\text{uniformly}} \frac{1}{2\pi i} \int_{S^1_{r,z_0}} \frac{f(w)}{w - z} dz$$

on $|z - w| \leq \frac{r}{2}$.

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holomorphic $\iff$ uniform limit of rationals

[Approximation of holomorphic functions on \$\mathbb{C}\$ by rational functions](#)

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And it has 2 siblings.

- stem
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