Info

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Classification of complex 1-manifold

uniformization

(Uniformization of simply connected Riemann surfaces) Every *non-compact simply connected* Riemann surface is biholomprhic to the complex plane $\mathbb{C}$ or the unit disk D. Every *compact simply connected* Riemann surface is biholomorphic to the Riemann sphere $\mathbb{C}P^1$

complex 1-manifolds structures on smooth 1-manifolds

homeomorphism class	moduli space	hol classes	universal covering	$\mathrm{AutMan}_{\mathbb{C}}$
plane $\mathbb{R}^2$	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(\mathbb{R}^2) = \{[\mathbb{C}], [$	$\mathbb{C}$	$\mathbb{C}$	
		disk/upper half plane D	D	$PSL(2)(\mathbb{R})$
(cylinder) $S^1 \times \mathbb{R} \cong$ (punctured plane) $\mathbb{R}^2 \setminus \{0\} \cong D \setminus \{(0,0)\}$	?	$\mathbb{C} \setminus \{0\}$	$\mathbb{C}$	$\mathbb{C}^\times$
		$D^* := D \setminus \{0\}$	D by exponential map	$SO(2)(\mathbb{R})$
		$D_r, r \in (0,1)$	D	??
sphere S^2	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(S^2) = \{[\mathbb{C}P^$	$\mathbb{C}P^1$	$\mathbb{C}P^1$	$PGL(2)(\mathbb{C})$
plane with $n > 1$ points removed	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(\mathbb{R}^2 \setminus \{0, 1, ..$		D	?
torus T^2	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(T^2) \cong SL(2)$	a rank 2 lattice $\Lambda \leq \mathbb{C}$ upto homothety	$\mathbb{C}$	depends on Λ
compact higher genus surfaces $T^2_g, g > 1$	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(T^2_g) \cong \mathbb{C}^{3g-3}$		D	finite of order $\leq 84(g-1)$
compact genus $g > 0$ surface with $n > 0$ points removed	$\underset{\mathrm{Man}_{\mathbb{C}}}{Md}(T^2_{g,n})$		D	
compact genus g surface with infinite points removed			D	
infinite genus				
...				

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- stem
  - Objects stemming from $\mathbb{R}$-spaces of dimension 2
    - Riemann surfaces
      - Classification of complex 1-manifold

And it has 16 siblings.

- stem
  - Objects stemming from $\mathbb{R}$-spaces of dimension 2
    - Riemann surfaces
      - Vector bundles on complex 1-manifolds
      - Classification of complex 1-manifold
      - Riemann's existence of holomorphic maps between compact Riemann surfaces
      - Divisors on compact Riemann surfaces
      - Fluid
      - Meromorphic differentials on Riemann surfaces
      - Holomorphic functions on a complex 1-manifold
      - Holomorphic maps of type $0 \xrightarrow{3} 0$
      - Rational maps
      - Hurwitz scheme
      - Moduli space of $\mathbb{C}$-manifolds
      - Meromorphic functions on a Riemann surface
      - Normalization of plane projective $\mathbb{C}$-curves and Riemann surfaces

- Riemann surfaces defined over a number field
- Hyperelliptic Riemann surfaces
- Riemann surfaces with a holomorphic 1-form