

Info

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Continuous functions on $\mathbb{R}$ with infinite limit are uniformly continuous

continuous function on $[a, \infty]$ is uniformly continuous

Let

$$f : [a, \infty) \rightarrow \mathbb{R}$$

be continuous such that

$$\lim_{x \rightarrow \infty} f(x) = f_\infty \in \mathbb{R}$$

exists. Then f is **uniformly continuous**.

Given $\epsilon > 0$, choose M large enough so that

$$x > M \implies |f(x) - f_\infty| < \frac{\epsilon}{2}$$

- On the **compact** interval $[a, N + 1]$ the **continuous** f is **uniformly continuous**. So there is a δ_1 such that

$$x, y \in [a, N + 1], |x - y| < \delta_1 \implies |f(x) - f(y)| < \epsilon$$

- Choose

$$\delta := \min\{\delta_1, 1\}$$

- Now, consider arbitrary $x, y \in [a, \infty)$ such that $|x - y| < \delta$.

- If $x, y \in [a, N + 1]$,

- On the **compact** interval $[a, N + 1]$ the **continuous** f is **uniformly continuous**. So there is a δ_1 such that

$$x, y \in [a, N + 1], |x - y| < \delta_1 \implies |f(x) - f(y)| < \epsilon$$

- If $x > N + 1$ then $|x - y| < \delta < 1 \implies y > N$ which means both $x, y \in [N, \infty)$

$$\begin{aligned} |f(x) - f(y)| &\leq |f(x) - f_\infty| + |f(y) - f_\infty| \\ &\leq \epsilon \end{aligned}$$

the converse is false and thus it is not a necessary condition

Example

$\sin x$ is uniformly continuous on $\mathbb{R}$ but infinite limits do not exist

Current note has 0 direct children and 0 total descendants.

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And it has 5 siblings.

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