Info

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is written (completely with human hands) by Rupadarshi Ray,
was created on January 26, 2026 4:46:36 PM,
was last modified on September 7, 2026 1:11:06 PM, and
was exported on September 7, 2026 3:09:57 PM.

# Connected circular open subsets of $\mathbb{C}^n$

## biholomorphisms from a bounded connected open circular

**(H Cartan)** Let $0 \in \Omega_1, \Omega_2 \subseteq \mathbb{C}^n$ be connected circular open subsets, $\Omega_1$ is bounded and

$$F : \Omega_1 \to \Omega_2, F(0) = 0$$

is a biholomorphism. Then $F$ is a linear transformation.

We have

$$F^{-1}(F(z)) = z$$
$$\implies \mathfrak{D}_0(F^{-1}) \circ \mathfrak{D}_0 F = \mathsf{Id}$$
$$\implies \mathfrak{D}_0(F^{-1}) = (\mathfrak{D}_0 F)^{-1}$$

- Now consider the holomorphic map

$$H_\theta : \Omega_1 \to \Omega_1$$
$$H_\theta(z) := F^{-1}(\exp(-i\theta) F(\exp(i\theta) z))$$

which satisfies

$$H(0) = 0, \mathfrak{D}_0 H = \mathsf{Id}$$

- By

**(H Cartan)** Let $\Omega \subseteq \mathbb{C}^n$ be a connected bounded open subset and

$$F : \Omega \to \Omega$$

be holomorphic. If for some $p \in \Omega$ we have $F(p) = p, \mathfrak{D}_p F = \mathsf{Id}$, then $F \equiv \mathsf{Id}_\Omega$.

$H \equiv \mathsf{Id}$.

- So

$$F(\exp(i\theta) z) = \exp(i\theta) F(z)$$

for every $\theta \in \mathbb{R}, z \in \Omega_1$.

- Thus, $F$ is linear.

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Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Connected circular open subsets of $\mathbb{C}^n$

And it has 39 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary
    - Complex ODEs in $\mathbb{C}$
    - Circle packing on $\mathbb{R}^2$
    - Circle packing converges to the Riemann biholomorphism
    - Bounded connected open subsets of $\mathbb{C}^n$
    - Connected circular open subsets of $\mathbb{C}^n$
    - Continuous functions on $\mathbb{R}^d$
    - Dyadic cubes
    - Curves
    - Differentiable functions
    - Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner transform
    - Harmonic functions composed with conformal maps
    - Hilbert transform
    - Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Holomorphic subsets of $\mathbb{C}^n$
    - Regular hypersurfaces in $\mathbb{R}^{2n}$
    - Orientable hypersurfaces in $\mathbb{R}^n$
    - Laplace operator on $\mathbb{R}^n$
    - $l^\infty(\mathbb{R})$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - Local $\mathbb{R}$-algebras
    - $l^p(\mathbb{R})$
    - Metric density of subsets of $\mathbb{R}^n$
    - Monotone functions on $\mathbb{R}$
    - Cauchy integral of periodic functions
    - Polygons with integer vertices
    - Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$

- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$