Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on October 18, 2023 1:52:49 PM,
was last modified on October 15, 2025 1:57:03 AM, and
was exported on September 7, 2026 3:31:11 PM.

Smooth Lie group action on a smooth manifold

$$\Phi : G \curvearrowright M$$

Definition. Smooth Lie group action on a smooth manifold

A (left) group action $\Phi : G \curvearrowright M$ by a Lie group G on a smooth manifold M

$$\Phi : G \times M \to M$$

is a **smooth Lie group action** if Φ is smooth.

We think of it as a "smooth" homomorphism

$$\Phi : G \to \mathrm{Diff}(M)$$

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induced Lie group actions

on the cotangent bundle

Let

$$G \curvearrowright M$$

be a smooth action, then this induces an action

$$\begin{aligned} G &\curvearrowright T^*M \\ (q, \eta) &\overset{\eta}{\mapsto} (gq, (\mathrm{d}g^{-1})^*_q \eta) \end{aligned}$$

and it commutes with the projection

$$\begin{array}{ccc} T^*M & \overset{g}{\to} & T^*M \\ \downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle \pi} \\ M & \overset{g}{\longrightarrow} & M \end{array}$$

so

$$\begin{aligned} \pi(gp) &= g\pi(p) \\ \implies \mathrm{d}_{gp}\pi \circ \mathrm{d}_p g &= \mathrm{d}_q g \circ \mathrm{d}_p \pi \\ \implies \mathrm{d}_p g^* \circ \mathrm{d}_{gp}\pi^* &= \mathrm{d}_p \pi^* \circ \mathrm{d}_q g^* \end{aligned}$$

Current note has 5 direct children and 5 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Smooth Lie group action on a smooth manifold
        - Smooth action by a compact Lie group
        - Generators of a Lie group action
        - Orbits of smooth Lie group actions
        - Proper Lie group actions
        - invariant Riemannian metric in a proper Lie group action

And it has 19 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Smooth Lie group action on a smooth manifold
      - Borel measures on a smooth manifold
      - G-bundles over smooth manifolds $G \curvearrowright P \to X$
      - Vector bundle on smooth manifolds
      - Curves in smooth manifolds
      - Density on smooth manifolds
      - Functions between smooth manifolds
      - Functions from smooth manifolds to $\mathbb{R}$
      - Flows on a smooth manifold
      - Locally $G \curvearrowright X$-manifolds
      - Smooth singular homology of a smooth manifold
      - Isomorphisms and automorphisms of smooth manifolds
      - Lagrangian flows
      - Local transformation rules
      - Moduli space of Riemannian metrics on a smooth manifold
      - Moduli space of quotients
      - Quotient manifolds
      - Submanifolds of smooth manifolds
      - Tangents on a smooth manifold