ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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Division in $\mathbb{Z}$

Let $a, b \in \mathbb{Z}, b > 0$ then there exists unique $q, r \in \mathbb{Z}$ such that $r \in [0, b)$

$$a = \underbrace{q}_{\text{quotient}} b + \underbrace{r}_{\text{remainder}}$$

Consider

$$S := \{a - xb \in \mathbb{N} | x \in \mathbb{Z}\}$$

- $$\begin{aligned} b &\geq 1 \\ \implies |a|b &\geq |a| \\ \implies a - (\underbrace{-|a|}_{x})b = a + |a|b &\geq a + |a| > 0 \\ \implies a - xb &\in S \end{aligned}$$

  Thus S is non-empty.

- $$r := \min\{a - xb \in \mathbb{N} | x \in \mathbb{Z}\}$$

  which exists by Well-Ordering principle

- As $r \in S$ we have

  $$r = a - qb$$

  for some $q \in \mathbb{Z}$.

Definition. Let $a, b \in \mathbb{Z}, a \neq 0$. Then b **is divisible by** a or a **divides** b if

$$b \in a\mathbb{Z}$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - $\mathbb{Z}$
    - Division in $\mathbb{Z}$

And it has 6 siblings.

- stretchy
  - $\mathbb{Z}$
    - Division in $\mathbb{Z}$
    - Euler totient function
    - $\gcd$ of integers
    - Primes in $\mathbb{N}$
    - Partitions of naturals
    - Prime-representing function