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Limit set of Fuchsian groups

for $\Lambda(\Gamma) \neq \partial \mathbb{R}H^n$

Theorem 12.2.5. *Let Γ be a nonelementary discrete subgroup of $M(B^n)$. Then the limit set $L(\Gamma)$ is perfect and is therefore uncountable.*

Theorem 12.2.11. *If P is a convex fundamental polyhedron for a discrete subgroup Γ of $M(B^n)$, then $O(P) = \bigcup \{g(\overline{P} \cap O(P)) : g \in \Gamma\}$.*

Let $\Gamma \leq O^+(1, n)(\mathbb{R}) \cong \text{Isom}(\mathbb{R}H^n) \curvearrowright \mathbb{R}H^n$ be a discrete subgroup. Then $\Lambda(\Gamma) \neq \partial \mathbb{R}H^n$ $\iff$ every convex fundamental polyhedron for Γ contains a half-space of $\mathbb{R}H^n$ $\iff$ Γ has a convex fundamental polyhedron that contains a half-space of $\mathbb{R}H^n$.

As a corollary we have

- Γ is of finite covolume $\implies$ convex fundamental polyhedron does not contain a half-space $\implies \Lambda(\Gamma) = \mathbb{R}H^n$

limit set of a non-elementary group agrees with the same for an infinite normal subgroup

Let $\Gamma \leq O^+(1, n)(\mathbb{R}) \cong \text{Isom}(\mathbb{R}H^n) \curvearrowright \mathbb{R}H^n$ be a discrete subgroup. If Γ is non-elementary and $H \trianglelefteq \Gamma$ is an infinite normal subgroup then $\Lambda(H) = \Lambda(\Gamma)$

As a consequence

- For

$$L \underbrace{\leq}_{\text{finite index}} \underbrace{H}_{\text{infinite}} \underbrace{\trianglelefteq}_{\text{infinite index}} \underbrace{\Gamma}_{\text{finite covolume}}$$

we have L is of infinite covolume, but $\Lambda(L) = \Lambda(H) = \Lambda(\Gamma) = \partial \mathbb{R}H^n$.

- In particular, **convex fundamental domain of L does not contain a half-plane but has infinite volume.**

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