> ⓘ Info
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Limit set of Fuchsian groups

for $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^n$

> **Theorem 12.2.5.** *Let be a nonelementary discrete subgroup of* $\mathrm{M}(B^n)$*. Then the limit set* $L(\ )$ *is perfect and is therefore uncountable.*

> **Theorem 12.2.11.** *If P is a convex fundamental polyhedron for a discrete subgroup of* $\mathrm{M}(B^n)$*, then* $O(\ ) = \cup\{g(\overline{P} \cap O(\ )) : g \in \ \}$.

> > Let
> > $$\Gamma \leq O^+(1,n)(\mathbb{R}) \cong \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n) \curvearrowright \mathbb{R}\boldsymbol{H}^n$$
> > be a discrete subgroup.
>
> Then $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^n$
> $$\Longleftrightarrow$$
> every convex fundamental polyhedron for Γ contains a half-space of $\mathbb{R}\boldsymbol{H}^n$
> $$\Longleftrightarrow$$
> Γ has a convex fundamental polyhedron that contains a half-space of $\mathbb{R}\boldsymbol{H}^n$.

As a corollary we have

- Γ is of finite covolume $\implies$ convex fundamental polyhedron does not contain a half-space $\implies$ $\Lambda(\Gamma) = \mathbb{R}\boldsymbol{H}^n$

limit set of a non-elementary group agrees with the same for an infinite normal subgroup

> > Let
> > $$\Gamma \leq O^+(1,n)(\mathbb{R}) \cong \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n) \curvearrowright \mathbb{R}\boldsymbol{H}^n$$
> > be a discrete subgroup.
>
> If Γ is non-elementary and $H \trianglelefteq \Gamma$ is an infinite normal subgroup then
> $$\Lambda(H) = \Lambda(\Gamma)$$

As a consequence

- For
$$L \underbrace{\leq}_{\text{finite index}} \underbrace{H}_{\text{infinite}} \underbrace{\lhd}_{\text{infinite index}} \underbrace{\Gamma}_{\text{finite covolume}}$$
we have L is of infinite covolume, but $\Lambda(L) = \Lambda(H) = \Lambda(\Gamma) = \partial \mathbb{R}\boldsymbol{H}^n$.
  - In particular, convex fundamental domain of L does not contain a half-plane but has infinite volume.
