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Tangent bundles on a complex manifold

We have

TM	$T^{\mathbb{C}}M := TM \otimes \mathbb{C}$	$T^{\mathbb{C}}M = \underbrace{T^{(1,0)}M}_{\ker i\mathrm{Id} - J \otimes \mathbb{C}} \oplus \underbrace{T^{(0,1)}M}_{\ker -i\mathrm{Id} - J \otimes \mathbb{C}}$
$J : TM \rightarrow TM$ $J^2 = -\mathrm{Id}_{TM}$	$J \otimes \mathbb{C} : T^{\mathbb{C}}M \rightarrow T^{\mathbb{C}}M$	
the usual tangent bundle of the smooth manifold M which is a $\mathbb{R}$ -vector bundle of $\mathbb{R}$ -rank $2n$ and when equipped with an almost complex structure J turns it into a $\mathbb{C}$ -vector bundle of $\mathbb{C}$ -rank n	is the complexified tangent bundle which is a $\mathbb{C}$ -vector bundle of $\mathbb{C}$ -rank $2n$	
$\mathbb{R} \left\{ \frac{\partial}{\partial x_k} \middle 1 \leq k \leq 2n \right\}$	$\mathbb{R} \left\{ \frac{\partial}{\partial x_k} \otimes 1, \frac{\partial}{\partial x_k} \otimes i \middle 1 \leq k \leq n \right\}$	$\mathbb{C} \left\{ \frac{\partial}{\partial z_k} \middle 1 \leq k \leq n \right\} \oplus \mathbb{C} \left\{ \frac{\partial}{\partial \bar{z}_k} \middle 1 \leq k \leq n \right\}$

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