Info

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Index of a vector field

Definition. Index of a vector field

(Poincaré-Hopf theorem) Let $v \in \Gamma TM$ be a vector field on a compact smooth manifold M. The index of the vector field equals the euler characteristic of M

$$\sum_{x \in \text{isolated-0}(v)} \text{index}_x(v) = \chi(M)$$

This gives on an orientable(?) manifold M, TM is a trivial bundle $\iff$ there is a non-vanishing vector field $\implies \chi(M) = 0$

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- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Tangent vector fields on a smooth manifold
            - Index of a vector field

And it has 4 siblings.

- structures based on sets
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