ⓘ Info

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Principal G-bundles over topological spaces

$$G \curvearrowright P \to X$$

Definition. Principal G-bundles $G \curvearrowright P \to X$

A **topological principal G-bundle over** X

$$G \curvearrowright P \to X := (G \curvearrowright P, \pi_P : P \to X, \mathscr{U})$$

where

- G is a Topological group called the structure group
- X is a T_2 topological space (Hausdorff?)
- a surjective continuous map

$$\pi_P : P \to X$$

- a right group action

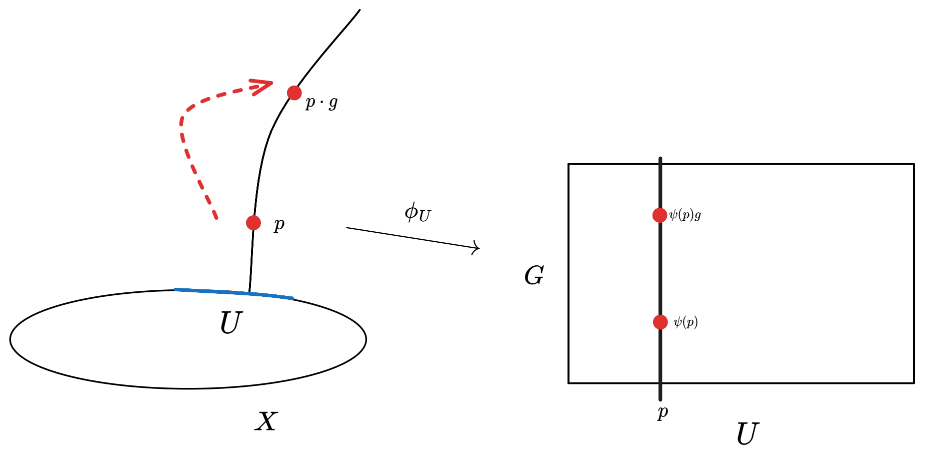
$$G \curvearrowright P$$

  - preserves fibres
- *locally trivial* for each $x \in X$, $\exists (U, \phi_U) \in \mathscr{U}$ where
  - U is a open nbd of x in X
  - and a homeomorphism

$$\begin{aligned} \phi_U : \pi_P^{-1}(U) &\to U \times G \\ p &\mapsto (\pi_P(p), \psi) \end{aligned}$$

  such that

$$\psi(p \cdot g) = \psi(p) g$$



For a principal G-bundle $G \curvearrowright P \to X$ we have

$$\begin{aligned} P/G &\cong X \\ p &\mapsto \pi(p) \end{aligned}$$

which means

$$\pi^{-1}(\pi(p)) = p \cdot G$$

Definition. The trivial G-bundle over X is defined as

$$\begin{aligned} \text{Trivial} : G \curvearrowright X \times G &\to X \\ (x, h) \overset{g}{\mapsto} (x, hg) & \\ (x, g) &\mapsto x \end{aligned}$$

The action $G \curvearrowright P$ is always free but generally not transitive.

classifying spaces

For each group G there is a $BG \in$ **Top** such that the isomorphism classes of G-bundles over any Top X is bijective to the homotopy classes of maps $X \to BG$

$$G\mathbf{Bun}_{\cong}(X) \cong \pi_0 \mathrm{Homeo}(X, BG)$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Principal G-bundles over topological spaces

$$G \curvearrowright P \to X$$

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology

- Principal G-bundles over topological spaces

$$G \curvearrowright P \to X$$

- Vector bundles over topological spaces
- Closure of a subset of topological space
- Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
- Covering dimension of topological spaces
- $\mathrm{End}\mathsf{Top}(X, Y)$
- Eilenberg–Steenrod functors

$$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$

- Local fields
- Local fields of characteristic 0
- Topological groups
- Continuous group action on a topological space
- Intersection of open, dense subsets of a topological space
- Local topological spaces
- Covering space of a topological space
- Borel measures on a topological space
- Convergence for nets in a topological space
- Loops and a open cover of a topological space
- Paths in a topological space
- Product of topological spaces
- Ringed topological spaces
- Sheaf on open sets of a topological space
- Closed subsets of a topological space
- Discrete subsets of topological spaces
- Types of topological spaces
- Locally compact Hausdorff spaces
- Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$