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Probability states and flows of an Hamiltonian system

[1]

$\mathbb{P}(M, \lambda_\omega)$ be the set of all probability measures. $\mathbb{P}^0(M, \lambda_\omega)$ be the set of all continuous probability measures $\mathcal{C}^0(M)\lambda_\omega$. $\mathbb{P}^\infty(M, \lambda_\omega)$ be the set of all continuous probability measures $\mathcal{C}^\infty(M)\lambda_\omega$.

Definition. Dynamics on $\mathbb{P}(M, \lambda_\omega)$ from Hamiltonian flow of H

Given a Hamiltonian flow $\Phi_H : \mathbb{R} \curvearrowright M$ on a symplectic manifold (M, ω), we have the flow on $\mathbb{P}(M, \lambda_\omega)$

$$\begin{aligned} \Phi_H : \mathbb{R} &\curvearrowright \mathbb{P}(X) \\ (t, \rho) &\mapsto \rho \circ \Phi_H^{-t} \end{aligned}$$

Definition.

$$\begin{aligned} \langle - \rangle_{(-)} : \mathcal{C}^\infty(M) \times \mathbb{P}(M, \lambda_\omega) &\to \mathbb{R} \\ (f, \rho) \mapsto \langle f \rangle_\rho &:= \int_{(M, \lambda_\omega)} f \end{aligned}$$

Definition. Shannon entropy

$$\begin{aligned} S_{\mathrm{Sh}} : \mathbb{P}(M, \lambda_\omega) &\to \mathbb{R} \\ \rho &\mapsto -\int_{(M, \lambda_\omega)} \rho \log(\rho) \end{aligned}$$

$$S_{\mathrm{Sh}}(\rho^{t_1}) = S_{\mathrm{Sh}}(\rho^{t_0})$$

Maximise the Shannon entropy

$\langle H \rangle_\rho \in \mathbb{R}$ be fixed, then S_{Sh} is stationary under variations of ρ

$$\iff$$

$\exists \beta \in \mathbb{R}$

$$\rho_\beta(x) = \frac{\exp(-\beta H(x))}{P(\beta)}$$

ρ_β remains invariant under the flow of H.

Definition. GIbbs manifold of a Hamiltonian system

Let $\mathcal{G}(M, \omega, H) \subseteq \mathbb{R}$ such that for all $\beta \in \mathcal{G}$ a Gibbs state ρ_β exists. So we have the functions

$$\mathcal{G} \to \mathbb{R}$$

$E(\beta) := \langle H \rangle_{\rho_\beta}$ and $S(\beta) := S_{\mathrm{Sh}}(\rho_\beta)$

$S, E \in \mathcal{C}^\infty(\mathcal{G}(M, \omega, H))$

$$S'(\beta) = \beta E'(\beta)$$

S, E are strictly decreasing functions on $\mathcal{G}$.

$$(S \circ E^{-1})'(x) = \frac{S'(E^{-1}(x))}{x} = E^{-1}(x)$$

Current note has 0 direct children and 0 total descendants.

And it has 5 siblings.

1. arXiv:2012.00582v2 [math.DG] 13 Jan 2021 ↩