Info

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S_n

Definition. S_n

Let $X := \{1, 2, \dots, n\}$ be the finite set with cardinality $n \geq 2$, then

$$S_n := \{f : X \to X : f \text{ is a bijection}\}$$

Then this set along with composition of functions

$$(S_X, \circ)$$

is defined to be the n-**th symmetric group**.

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is a group and every element can be written as a product of disjoint cycles.

- Irreducible representations of S_n

center

$$Z(S_n) = 0$$

for all $n \geq 3$

Finite cycle decomposition

conjugacy classes

$$\tau\sigma\tau^{-1}$$

$$\tau(\sigma_1 \dots \sigma_n)\tau^{-1}$$

$$= (\tau(\sigma_{1,1})\,\tau(\sigma_{1,2})\ \dots\ )(\tau(\sigma_{2,1})\,\tau(\sigma_{2,2})\ \dots\ )\dots$$

For $n \geq 1$, each conjugacy class of $S_n \leftrightarrow$ a cycle type, that is for any $k \leq n$

$$(\lambda_1, \dots, \lambda_k) \in \mathbb{N}^k, \sum_{i=1}^{k} \lambda_i = n, \lambda_1 \geq \lambda_2 \geq \dots$$

in $S_n \leftrightarrow$ to a *partition of the natural number* $n \in \mathbb{N}$.

- Representations of S_n
  - S_5

	number of elements in the conjugacy class
A Partitions of naturals $\lambda_1, \dots$	$\dfrac{n!}{\prod_j (j)^{\lambda_j} \lambda_j!}$ [1]

Current note has 2 direct children and 3 total descendants.

- stretchy
  - Symmetric group
    - S_n
      - Finite cycle decomposition
      - Representations of S_n
        - Irreducible representations of S_n

And it has 5 siblings.

- stretchy
  - Symmetric group
    - S_3
    - S_4
    - S_5
    - S_n
    - $\lim S_n$

1. Conjugacy class size formula in symmetric group - Groupprops (subwiki.org) ↩