Info

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Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$

Definition. Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$

A vector space X over $k \in \{\mathbb{R}, \mathbb{C}\}$ is a **topological k-vector space** if there is a T_1 topology on X such that

$$\begin{aligned} + : X \times X &\to X \\ k \times X &\to X \end{aligned}$$

are continuous.

Definition. Vector bounded subsets of topological vector spaces

A subset E of a topological vector space is **vector bounded** if for every nb V of $0_X \in X$ there is a $s(V) > 0$ such that

$$\forall t > s(V), E \subseteq tV$$

> **Proposition:** Let X be a topological vector space. Then translations and scaling are homeomorphisms of X onto X.

Definition. Types of topological vector spaces

X is	if
locally convex	there is a local base of convex subsets
locally vector bounded	0_X has a bounded onb
locally compact	0_X has a onb whose closure is compact
metrizable	there is a metric d which induces the topology on X
F-space	there is an translation invariant complete metric d which induces the topology on X
Frechet space	X is a locally convex F-space
normable	there exists a norm which induces the topology on X
Banach space	there exists a norm which induces a complete metric, and which induces the topology on X
vector Heine-Borel	every closed and vector bounded subset of X is compact

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Current note has 7 direct children and 11 total descendants.

- structures based on sets
  - Topological spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$
      - Banach spaces
        - Dual of Banach spaces
      - Dual space of a topological vector space
      - Balls in dual of a topological vector space
      - Convex compact subsets of weak* dual
      - Weak* topology on dual of a topological vector space
      - Compact endomorphisms
      - map
        - Image of balls under linear map between topological vector spaces
        - Limit of a sequence of linear maps
        - Uniform boundedness of a famly of linear maps
      - Weak topology on a topological vector space

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology
    - Principal G-bundles over topological spaces

      $$G \curvearrowright P \to X$$

    - Vector bundles over topological spaces
    - Closure of a subset of topological space
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
    - Covering dimension of topological spaces
    - $\mathrm{End}\mathsf{Top}(X, Y)$
    - Eilenberg–Steenrod functors

      $$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$

- Local fields
- Local fields of characteristic 0
- Topological groups
- Continuous group action on a topological space
- Intersection of open, dense subsets of a topological space
- Local topological spaces
- Covering space of a topological space
- Borel measures on a topological space
- Convergence for nets in a topological space
- Loops and a open cover of a topological space
- Paths in a topological space
- Product of topological spaces
- Ringed topological spaces
- Sheaf on open sets of a topological space
- Closed subsets of a topological space
- Discrete subsets of topological spaces
- Types of topological spaces
- Locally compact Hausdorff spaces
- Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$