> **Info**
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
> was created on December 21, 2024 1:25:04 PM,
> was last modified on September 7, 2026 1:29:48 PM, and
> was exported on September 7, 2026 2:50:34 PM.

#function/real/1

$$\frac{\cos x^k}{x}$$

- $$\begin{aligned} \mathbb{R} \setminus \{0\} &\to \mathbb{R} \\ x &\mapsto \frac{\cos x^k}{x} \end{aligned}$$

  is a smooth (analytic) function which is bounded on $[a, \infty)$ for $a > 0$.

- $$\begin{aligned} \left| \frac{\cos x^k}{x} \right| &\leq \frac{1}{x} \text{ on } [1, \infty) \\ \implies \left| \frac{\cos x^k}{x} \right| &\xrightarrow{x \to \infty} 0 \end{aligned}$$

  - By

    > Let
    >
    > $$f : [a, \infty) \to \mathbb{R}$$
    >
    > be continuous such that
    >
    > $$\lim_{x \to \infty} f(x) = f_\infty \in \mathbb{R}$$
    >
    > exists. Then f is **uniformly continuous**.

    it is **uniformly continuous** on $[1, \infty)$.

  - On $(0, 1)$

    $$\left| \frac{\cos x^k}{x} \right| \geq \frac{1}{x}$$

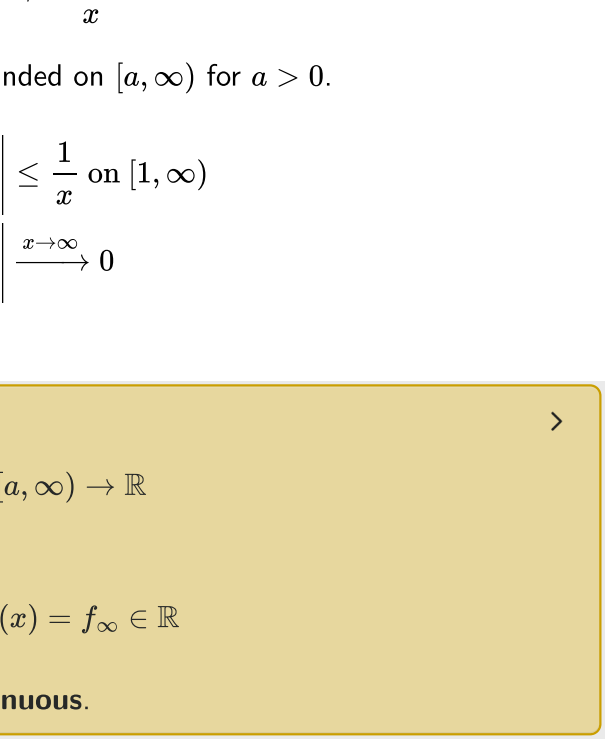
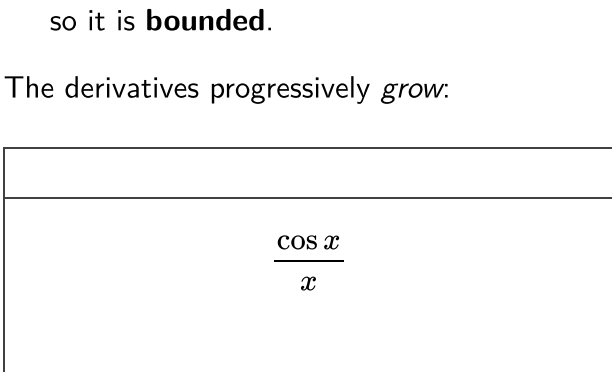
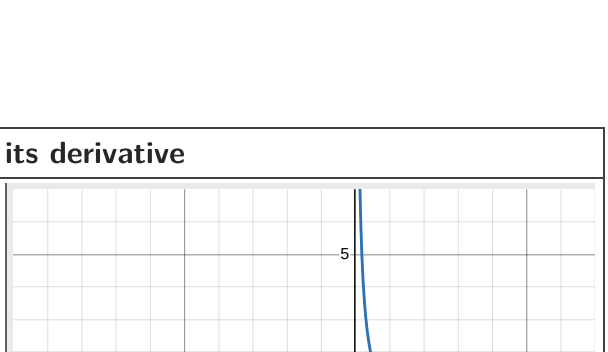
    is **unbounded**.

  - On $[1, \infty)$

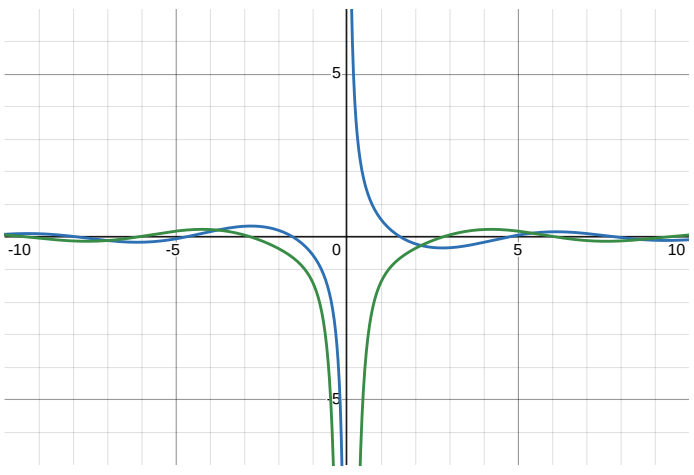
    $$\left| \frac{\cos x^k}{x} \right| \leq 1$$

    so it is **bounded**.

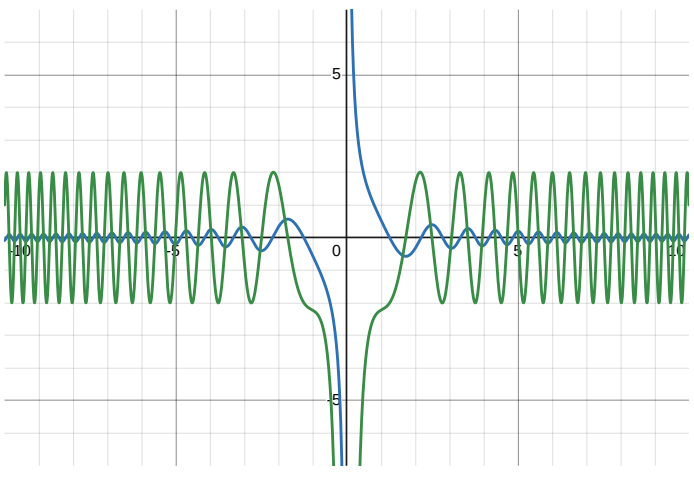
The derivatives progressively *grow*:

	its derivative
$\frac{\cos x}{x}$	
$\frac{\cos x^2}{x}$	
$\frac{\cos x^3}{x}$ 	• $\frac{\mathrm{d}}{\mathrm{d}x} \frac{\cos x^3}{x} = -3x \sin(x^3) - \frac{\cos(x^3)}{x^2}$ 

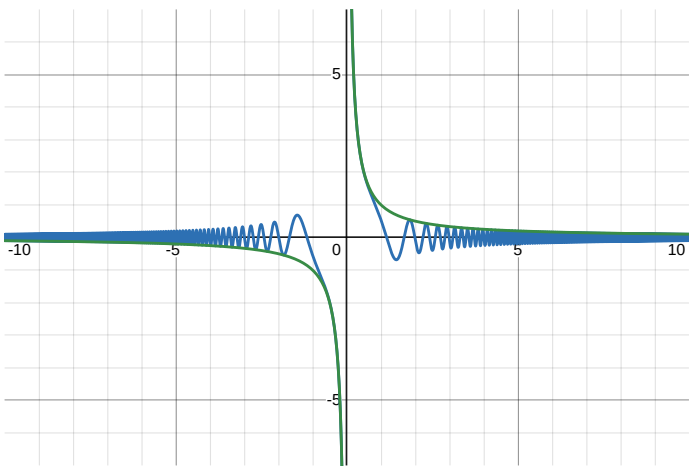
$$\frac{\cos x}{x}$$



$$\frac{\cos x^2}{x}$$

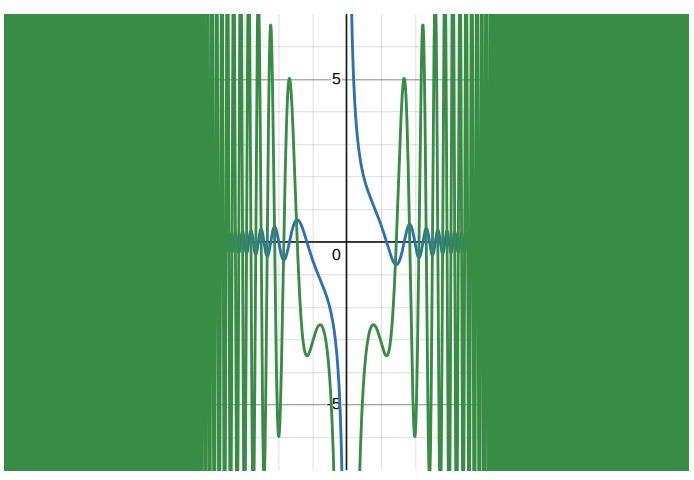


$$\frac{\cos x^3}{x}$$



derivative

- $$\frac{\mathrm{d}}{\mathrm{d}x}\frac{\cos x^3}{x} = -3x\sin(x^3) - \frac{\cos(x^3)}{x^2}$$



- On $[1,\infty)$ it is **unbounded** as for $x^3 = 2n\pi + \frac{\pi}{2}$

$$-3x\underbrace{\sin(x^3)}_{1} - \frac{1}{x^2}\underbrace{\cos(x^3)}_{0} = -3\left(2n\pi + \frac{\pi}{2}\right)$$

> - So $\cos(x^3)/x$ is a **bounded, uniformly continuous** function whose derivative is **unbounded**.

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $\frac{\cos x^k}{x}$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $\cos\sqrt{z}$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $\frac{r_0}{1+\epsilon\cos b\theta}$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}$[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.]]

- $\int \frac{1}{x^2+bx+c}$
- $\frac{\exp(z)}{1-z}$
- $\sin x$
- $\sin(x \sin(x))$
- $\frac{\sin x}{x}$
- stock.sin x by xk
- $\sin x^2$
- $\sqrt{z}$
- $\sqrt{z^2-1}$
- Theta functions
- Weierstrass elliptic function $\wp$
- Weierstrass' primary factors
- $x^2 \sin \frac{1}{x}$
- $x + x^2 \sin \frac{1}{x}$