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$$GL(2)(\mathbb{R}) / GL(2)(\mathbb{Z})$$

Proposition:

$$GL_+(2,\mathbb{R}) / SL(2)(\mathbb{Z}) \cong GL(2)(\mathbb{R}) / GL(2)(\mathbb{Z})$$

quotients of $L := GL(2)(\mathbb{R}) / GL(2)$	homeo	moduli of lattices/flat tori	$\mathbb{Z}$ -basis
L	4-manifold?	all lattices/tori	
$\frac{L}{O_+(2,\mathbb{R})}$	$\mathbb{R}^3$	upto oriented isometries	
$\frac{L}{CO_+(2,\mathbb{R})} \cong? \frac{H}{SL(2)(\mathbb{Z})}$	$\mathbb{C} \cong \mathbb{R}^2$	upto oriented conformal maps	$v_1 = (1,0)$ and $v_2 \in F$
$\frac{L}{O(2,\mathbb{R})}$	$[0,\infty) \times \mathbb{R}^2?$	upto isometries	12-5. CLASSIFICATION OF FLAT TORI: Let $T^2 = \mathbb{S}^1 \times \mathbb{S}^1$ denote the 2-torus. Show that if g is a flat Riemannian metric on T^2, then (T^2,g) is isometric to one and only one Riemannian quotient $\mathbb{R}^2/\Lambda$, where Λ is a lattice generated by a basis of the form $\{(a,0),(b,c)\}$, with $a > 0, 0 \leq b \leq a/2, c > 0$, and $b^2 + c^2 \geq a^2$. [Hint: Given a lattice $\Lambda \subseteq \mathbb{R}^2$, let v_1 be an element of $\Lambda \setminus \{0,0\}$ of minimal norm; let v_2 be an element of $\Lambda \setminus \langle v_1 \rangle$ of minimal norm (where $\langle v_1 \rangle$ is the cyclic subgroup generated by v_1), chosen so that the angle between v_1 and v_2 is less than or equal to $\pi/2$, and then apply a suitable orthogonal transformation.] [1]
$\frac{L}{CO(2,\mathbb{R})} \cong? \frac{H/SL(2)}{\mathbb{Z}_2}$	$[0,\infty) \times \mathbb{R}?$	upto (unoriented) conformal maps	1/22 (if not $a_1 = a_2$ would be shorter than a_1). The class of the lattice Γ is hence determined by the position of a_2 in the halford domain. $M = \{(x,y)/x^2 + y^2 \geq 1, 0 \leq x \leq 1/2, y > 0\}$, and two lattices corresponding to two different points of M belong to two different classes.

[2]

[3]

Current note has 0 direct children and 0 total descendants.

- [squishy](#)
 - GL
 - $GL(2)(\mathbb{R}) / GL(2)(\mathbb{Z})$

And it has 1 siblings.

- [squishy](#)
 - GL
 - $GL(2)(\mathbb{R}) / GL(2)(\mathbb{Z})$

1. <https://www.desmos.com/calculator/l6fgnoug6> ↩
2. <matematicas.unex.es/~extracta/Vol-25-2/25J2Ramir.pdf#page=24.95> ↩
3. [Action of \$GL_2\(\mathbb{R}\)\$ on the space of lattices - Mathematics Stack Exchange](#) ↩