Info

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$$z^3 - 3z$$

```
from script_complexfunccolor import *

ab = (-10,10)
cd = (-5, 5)

width = 10
height = width * (cd[1] - cd[0]) / (ab[1] - ab[0])

plt.figure(figsize=(width, height))
f = lambda z: z**3-3*z
plot_domain(domaincol_m, f, re=ab, im=cd, title='$f(z)=z^3-3z$',
daxis=True)
plt.tight_layout()
plt.show()
```

The map

$$\begin{aligned} \mathbb{C}P^1 &\to \mathbb{C}P^1 \\ z &\mapsto z^3 - 3z \\ \infty &\mapsto \infty \end{aligned}$$

is degree 3, so it is a 3-sheeted branched cover.

The ramification points are at $z = \infty$ and

$$z : f'(z) = 3z^2 - 3 = 0 \iff z = -1, 1$$

and the branch points

$$\begin{aligned} f(\infty) &= \infty \\ f(-1) &= 2 \\ f(1) &= -2 \end{aligned}$$

The preimage of the branch points are

$$f^{-1}(\infty) = \{\infty\}$$

and

$$(f')^{-1}(f(-1)) =$$

Up to affine change of coordinates on domain and range, every degree 3 polynomial is equivalent to z^3 or $z^3 - 3z$

A general polynomial degree 3 polynomial in $\mathbb{C}[z]$, when scaled to be monic, is

$$f(z) = z^3 + az^2 + bz + c$$

- By $z \mapsto z + \frac{a}{3}$, f can be turned into

$$f\left(z - \frac{a}{3}\right) = z^3 + pz + q$$

- by subtracting the constant

$$f\left(z - \frac{a}{3}\right) = -q = z^3 + pz$$

- for $p = 0$ we have z^3
- for $p \neq 0$, we scale $z \mapsto sz$ such that $p = -3s^2$ we have

$$f\left(sz - \frac{a}{3}\right) = -q = s^3(z^3 - 3z)$$

thus

$$\frac{1}{s^3} f\left(sz - \frac{a}{3}\right) = z^3 - 3z$$

Deck transformation group is trivial

[1]

[2]

[3]

[4]

Inverse

> **Proposition:**
>
> $$2w = z^3 - 3z$$
>
> implies
>
> $$z = \sqrt[3]{w + \sqrt{w^2 - 1}} + \frac{1}{\sqrt[3]{w + \sqrt{w^2 - 1}}}$$

[5]

The induced map on fundamental groups

[6]

The induced field extension of $\mathbb{C}(z)$

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $z^3 - 3z$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $\cos \sqrt{z}$
    - The complex logarithm
    - $\sum_{n \geq 0} z^{n!}$
    - $z^3 - 3z$
    - $\frac{r_0}{1 + \epsilon \cos b\theta}$
    - $\cos\left(\pi \frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.]]$
    - $\int \frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x \sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2 - 1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2 \sin \frac{1}{x}$
    - $x + x^2 \sin \frac{1}{x}$

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1. https://math.stackexchange.com/a/4620481/1290493 ↩
2. https://math.stackexchange.com/q/5038875/1290493 ↩
3. An investigation of certain Riemann surfaces ↩
4. 1904.07689 ↩
5. https://www.jstor.org/stable/2307507 Deduction of Cardano's Formula by Conformal Mapping ↩
6. https://math.stackexchange.com/q/4620770/1290493 ↩