Info

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Boundary of CAT(0) metric spaces

Definition. Asymptotic geodesic rays

Let X be a complete CAT(0) metric space. Two minimal geodesic rays c_1, c_2 are said to be **asymptotic** if there exists a $K > 0$ such that

$$\forall t \geq 0, d(c_1(t), c_2(t)) \leq K$$

Proposition:

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is an equivalence relation.

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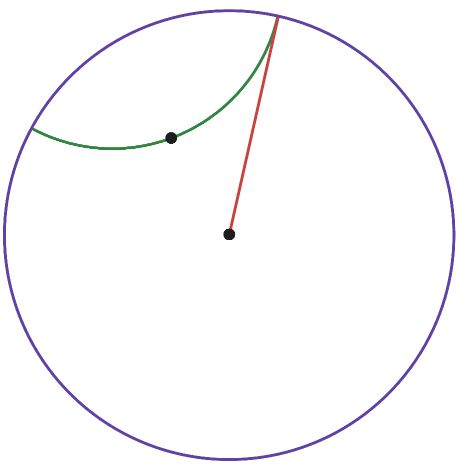
is called the **boundary of** X

$$\partial X := \frac{\{c : [0, \infty) \to X \text{ minimal geodesic rays}\}}{\text{asymptotic}}$$

Proposition: Let X be a complete CAT(0) space and $c : [0, \infty) \to X$ is a geodesic ray starting at $c(0) = x \in X$. Then for every point $x' \in X$ there **exists an unique** geodesic ray c' which starts at $c'(0) = x'$ and is **asymptotic** to c.



This geodesic ray c' shall be denoted $\overline{x'\xi}$ for $\xi := [c(0)] \in \partial X$.

Proposition: Given $\epsilon, a, s > 0$ $\exists T(\epsilon, a, s) > 0$ such that: for any

- $x, x' \in X$ with $d(x, x') = a$
- c is a minimal geodesic ray starting at $c(0) = x$
  consider the minimal geodesic ray σ_t joining $x' = \sigma_t(0)$ to $c(t)$. Then

$$\forall t \geq T(\epsilon, a, s), t' > 0, d(\sigma_t(s), \sigma_{t+t'}(s)) < \epsilon$$

isometries extend to $\overline{X}$