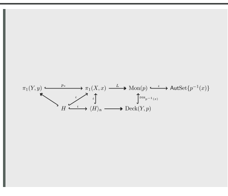
ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on February 20, 2024 12:06:48 PM,
was last modified on September 6, 2026 11:24:53 PM, and
was exported on September 7, 2026 3:24:51 PM.

Correspondence between covering spaces and subgroups of the fundamental group

Let X just have a universal covering space? or path-connected, semi-locally simply connected required?

covering space (Y,y)	$p_*\pi_1(Y,y) =: H \le$	**surjective homomorphism to Deck group**	**lift homomorphism from fundamental group to Monodromy group**	**restriction homomorphism from Deck group to Monodromy group**
any covering spaces (possibly not path-connected)			homomorphism $\pi_1(X,x) \to \mathrm{AutS}$ whose image is defined as $\mathrm{Mon}(p)$	
n-sheeted covering spaces (possibly not path-connected)			conjugacy class of homomorphisms $\pi_1(X,x) \to S_n$ for n upto?	
if surjective then cover is path connected?			classes of **surjective** homomorphisms (cannot happen when $\pi_1(X,x)$ is Abelian for example)	
path-connected *based* covering spaces $p:(Y,y)\to(X,x$ that induces $p_*:\pi_1(Y,y)\hookrightarrow \pi$	subgroups $H := p_*\pi_1(Y,y)$ of $\pi_1(X,x_0)$	surjective homomorphism $\mathrm{N}(H) \to \mathrm{Deck}(Y,$ whose kernel is H that gives us $\frac{\mathrm{N}(H)}{H} \cong \mathrm{Deck}(Y$	$\pi_1(X,x) \to \mathrm{AutS}$	$\mathrm{res}:\mathrm{Deck}(Y,p) -$ is an *injective* group homomorphism. There *exists an unique* Deck transformation between y_1, y if $p_*\pi_1(Y,y_1) = p_*\pi$
	by *covering* group action through Deck transformations $\frac{U(X,x)}{H}$			
path-connected covering spaces (ignoring base points)	conjugacy classes of subgroups of $\pi_1(X,x)$			
$Y_1 \to Y_2 \to X$	$H_1 \le H_2 \le \pi_1(X$			
normal covering spaces p of X that induces $p_*:\pi_1(Y,y)\hookrightarrow \pi$	*normal* subgroups $H = p_*\pi_1(Y,y)$ of $\pi(X,x_0)$	surjective homomorphism $\pi_1(X,x) \to \mathrm{Deck}($ whose kernel is $p_*\pi_1(Y,y)$ that gives us $\frac{\pi_1(X,x)}{H} \cong \mathrm{Deck}$	$\pi_1(X,x) \to \mathrm{AutS}$ is a group homomorphism whose kernel is H, hence $\frac{\pi_1(X,x)}{H} \cong \mathrm{Mon}$	$\mathrm{res}:\mathrm{Deck}(Y,p) \cong$ There exists a Deck transformation between any pair of elements in $p^{-1}(x)$.
	quotients by properly discontinuous *transitive* group action through Deck transformations			
n-cyclic cover	normal subgroup H such that $\frac{\pi_1(X,x)}{H} \cong \mathbb{Z}_n$	$\mathrm{Dec}(Y,p) \cong \mathbb{Z}_n$		

covering space (Y, y)	$p_*\pi_1(Y, y) =: H \leq$	**surjective homomorphism to Deck group**	**lift homomorphism from fundamental group to Monodromy group**	**restriction homomorphism from Deck group to Monodromy group**
infinite cyclic cover	normal subgroup H such that $\frac{\pi_1(X,x)}{H} \cong \mathbb{Z}$	$\mathrm{Dec}(Y, p) \cong \mathbb{Z}$		
Abelian covering spaces	*normal* subgroups that *contain the commutator subgroup*	$\frac{\pi_1(X,x)}{H} \cong \mathrm{Deck}$ becomes an Abilean group!		$\mathrm{res} : \mathrm{Deck}(Y, p) \cong$ is an isomorpshism of Abelian groups
Universal Abelian covering space	$H = [\pi_1(X, x), \pi_1$ the commutator subgroup	$\frac{\pi_1(X,x)}{H} \cong \mathrm{Deck}$ becomes the largest Abilean (as a quotient) group!		$\mathrm{res} : \mathrm{Deck}(Y, p) \cong$ is an isomorpshism of Abelian groups
the *universal cover* whose fundamental group is 0	the subgroup is 0			
	$\frac{U(X,x)}{0}$			

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Covering space of a topological space
      - Correspondence between covering spaces and subgroups of the fundamental group

And it has 1 siblings.

- structures based on sets
  - Topological spaces
    - Covering space of a topological space
      - Correspondence between covering spaces and subgroups of the fundamental group