ⓘ Info

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Loops and a open cover of a topological space

Let $U, V \subseteq X$ be two open sets. Then we have the inclusion maps

$$\begin{tikzcd} & U \cup V & \\ U \arrow[ur, "\iota_U"] & & V \arrow[ul, "\iota_V"'] \\ & U \cap V \arrow[ul, "\iota_{U\cap V,U}"] \arrow[ur, "\iota_{U\cap V,V}"'] & \end{tikzcd}$$

which gives

$$\begin{tikzcd} & \pi_1(U \cup V) & \\ \pi_1(U) \arrow[ur, "\iota_U^*"] & & \pi_1(V) \arrow[ul, "\iota_V^*"'] \\ & \pi_1(U \cap V) \arrow[ul, "\iota_{U\cap V,U}^*"] \arrow[ur, "\iota_{U\cap V,V}^*"'] & \end{tikzcd}$$

Notice

- injective inclusion maps do not functor to injective homomorphisms, two homotopy classes of paths may become a single class after pushforward

path-connected cover with path-connected intersections

We consider the case when the two open sets U, V are path-connected and $U \cap V$ is path connected.

Let (X, x_0) is a topological space with basepoint with

$$X = \bigcup_\alpha A_\alpha$$

where A_α are path connected open sets each containing the base point $x_0 \in X$ and $A_\alpha \cap A_\beta$ is path-connected

$$\implies$$

for any $[f] \in \pi_1(X)$ can be **factored** $[f] = [a_1^f] \dots [a_m^f]$ such that each $[a_i^f]$ is a class of loop in some A_α

$$\implies$$

from the inclusions $i_\alpha : A_\alpha \to X$ the extended induced map

$$\star_\alpha i_\alpha : \star_\alpha A_\alpha \to X$$

is **surjective**.

path-connected cover with path-connected single and double intersections

Let (X, x_0) is a topological space with basepoint with

$$X = \bigcup_\alpha A_\alpha$$

where A_α are path connected open sets each containing the base point $x_0 \in X$, $A_\alpha \cap A_\beta$ is path-connected and $A_\alpha \cap A_\beta \cap A_\delta$ is path-connected

$$\implies$$

from the inclusions $i_\alpha : A_\alpha \to X$ the kernel of the extended induced map

$$\ker(\star_\alpha i_\alpha : \star_\alpha A_\alpha \to X)$$

is the normal subgroup generated by the elements of the form $i_{\alpha\beta}(\omega) i_{\beta\alpha}(\omega)^{-1}$ for $\omega \in \pi_1(A_\alpha \cap A_\beta)$.

in the small category of cover

(Groupoidal van Kampen) Let $\mathfrak{U}$ be an open cover of X by path connected open subsets which is closed under finite intersections. We regard $\mathfrak{U}$ as a small category with inclusion of subsets as morphisms which makes

$$\Pi\Big|_{\mathfrak{U}} : \mathfrak{U} \to \mathsf{Grd}$$

a diagram in groupoids. Then the **groupoid** $\Pi(X)$ **is the colimit of this diagram**

$$\Pi(X) \cong \mathrm{colim}_{U \in \mathfrak{U}} \Pi(U)$$

We have the universal property of colimits of diagrams.

universal property

> **Proposition:** Kernel is a diagram
>
> $$K \xrightarrow{k_f} M \xrightarrow{f} N$$
>
> such that $f \circ k_f = 0$ and is universal for this property.

Current note has 1 direct children and 1 total descendants.

- structures based on sets
  - Topological spaces
    - Loops and a open cover of a topological space
      - Factorization of loops in a topological space with path-connected cover with path-connected intersections

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology
    - Principal G-bundles over topological spaces

$$G \curvearrowright P \to X$$

    - Vector bundles over topological spaces
    - Closure of a subset of topological space
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
    - Covering dimension of topological spaces
    - $\mathrm{End}\mathsf{Top}(X, Y)$
    - Eilenberg–Steenrod functors

$$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$

    - Local fields
    - Local fields of characteristic 0
    - Topological groups
    - Continuous group action on a topological space
    - Intersection of open, dense subsets of a topological space
    - Local topological spaces
    - Covering space of a topological space
    - Borel measures on a topological space
    - Convergence for nets in a topological space
    - Loops and a open cover of a topological space
    - Paths in a topological space
    - Product of topological spaces
    - Ringed topological spaces
    - Sheaf on open sets of a topological space
    - Closed subsets of a topological space
    - Discrete subsets of topological spaces
    - Types of topological spaces
    - Locally compact Hausdorff spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$