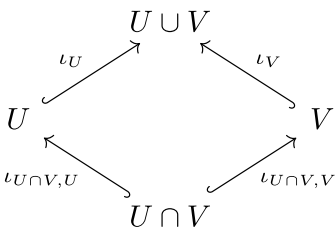


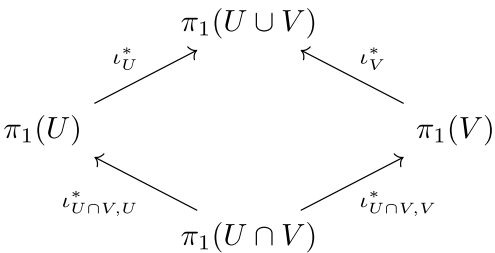
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Loops and a open cover of a topological space

Let $U, V \subseteq X$ be two open sets. Then we have the inclusion maps



which gives



Notice

- injective inclusion maps do not functor to injective homomorphisms, two homotopy classes of paths may become a single class after pushforward

path-connected cover with path-connected intersections

We consider the case when the two open sets U, V are path-connected and $U \cap V$ is path connected.

☰

Let (X, x_0) is a topological space with basepoint with

>

$$X = \bigcup_{\alpha} A_{\alpha}$$

where A_{α} are path connected open sets each containing the base point $x_0 \in X$ and $A_{\alpha} \cap A_{\beta}$ is path-connected

$$\implies$$

for any $[f] \in \pi_1(X)$ can be **factored** $[f] = [a_1^f] \dots [a_m^f]$ such that each $[a_i^f]$ is a class of loop in some A_{α}

$$\implies$$

from the inclusions $i_{\alpha} : A_{\alpha} \rightarrow X$ the extended induced map

$$\star_{\alpha} i_{\alpha} : \star_{\alpha} A_{\alpha} \rightarrow X$$

is **surjective**.

path-connected cover with path-connected single and double intersections

☰

Let (X, x_0) is a topological space with basepoint with

>

$$X = \bigcup_{\alpha} A_{\alpha}$$

where A_{α} are path connected open sets each containing the base point $x_0 \in X$, $A_{\alpha} \cap A_{\beta}$ is path-connected and $A_{\alpha} \cap A_{\beta} \cap A_{\delta}$ is path-connected

$$\implies$$

from the inclusions $i_{\alpha} : A_{\alpha} \rightarrow X$ the kernel of the extended induced map

$$\ker(\star_{\alpha} i_{\alpha} : \star_{\alpha} A_{\alpha} \rightarrow X)$$

is the normal subgroup generated by the elements of the form $i_{\alpha\beta}(\omega)i_{\beta\alpha}(\omega)^{-1}$ for $\omega \in \pi_1(A_{\alpha} \cap A_{\beta})$.

in the small category of cover

☰

(Groupoidal van Kampen) Let $\mathfrak{U}$ be an open cover of X by path connected open subsets which is closed under finite intersections. We regard $\mathfrak{U}$ as a small category with inclusion of subsets as morphisms which makes

$$\Pi \Big|_{\mathfrak{U}} : \mathfrak{U} \rightarrow \mathbf{Grd}$$

a [diagram in](#) groupoids. Then the **groupoid $\Pi(X)$ is the colimit of this diagram**

$$\Pi(X) \cong \operatorname{colim}_{U \in \mathfrak{U}} \Pi(U)$$

We have the universal property of colimits of diagrams.

universal property

Proposition:

Kernel is a diagram

$$K \xrightarrow{k_f} M \xrightarrow{f} N$$

such that $f \circ k_f = 0$ and is universal for this property.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Loops and a open cover of a topological space](#)
 - [Factorization of loops in a topological space with path-connected cover with path-connected intersections](#)

And it has 29 siblings.

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 - [Discrete subgroup acting on coset space](#)
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$$G \curvearrowright P \rightarrow X$$
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