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Holomorphic functions meromorphic at infinity

Definition. Holomorphic functions meromorphic at infinity

A holomorphic function

$$f : \{z \in \mathbb{C} \mid |z| > R\} \rightarrow \mathbb{C}$$

is **meromorphic at** ∞ for all $M > 0$ there is a $r \in (R, \infty)$ such that

$$|z| > r \implies |f(z)| > M$$

- Equivalently

$$\begin{aligned} D^*_{\frac{1}{R}} &\rightarrow \mathbb{C} \\ w &\mapsto f\left(\frac{1}{w}\right) \end{aligned}$$

is meromorphic at $w = 0$

Definition. Meromorphic function on the complex plane

A holomorphic function

$$f : z_0 + D^*_R \rightarrow \mathbb{C}$$

is **meromorphic at** z_0 if

$$|f(z)| \xrightarrow{z \rightarrow z_0} \infty$$

which means

$$\forall M > 0 \exists r \in (0, R) : |z - z_0| < r \implies |f(z)| > M$$

- By

- Thus for any function f that is meromorphic at z_0 , there is a holomorphic $h : z_0 + D_r \rightarrow \mathbb{C}$ and positive integer k called the **order of the pole at** z_0 such that

$$f(z) = \frac{h(z)}{(z - z_0)^k} \text{ on } z_0 + D^*_r$$

Moreover $h(z_0) \neq 0$.

we have for all $w \in D^*_r$

$$f\left(\frac{1}{w}\right) = \frac{h(w)}{w^k} = f(z) = z^nh\left(\frac{1}{z}\right)$$

where h is holomorphic on some D_r , $h(0) \neq 0$ and k is a positive integer called the **order of the pole of f at infinity**.

- [Non constant entire functions are meromorphic at infinity.](#)

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And it has 19 siblings.

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