Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on May 21, 2022 6:44:50 AM,
was last modified on May 24, 2026 6:43:12 PM, and
was exported on September 7, 2026 3:09:54 PM.

Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form

Definition. Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form

For $H \in \mathcal{C}^1(\mathbb{R}^{2n})$, the hamiltonian vector field in $\mathbb{R}^{2n}$ with coordinates (q_i, p_i) is

$$\dot{q} = \frac{\partial H}{\partial p}$$
$$\dot{p} = -\frac{\partial H}{\partial q}$$

which we can write as

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial q} \end{bmatrix}$$

which on $\mathbb{C}$

$$[\dot{p} + i\dot{q}] = i\left[\frac{\partial H}{\partial p} + i\frac{\partial H}{\partial q}\right]$$

On $\mathbb{C}^n$, making $z_i := p + iq$ this becomes

$$[\dot{z}_i] = \begin{bmatrix} i & & & & \\ & i & & & \\ & & i & & \\ & & \dots & & \\ & & & & i \end{bmatrix} \left[\frac{\partial H}{\partial p_i} + i\frac{\partial H}{\partial q_i}\right]$$

$$\dot{z}_i = i\left(\frac{\partial H}{\partial p_i} + i\frac{\partial H}{\partial q_i}\right)$$

or

$$\dot{z}_i = 2i\frac{\partial H}{\partial \bar{z}_i}$$

more compactly.

On $n = 1$, we have

$$\frac{\dot{z}}{2i} = \frac{\partial H}{\partial \bar{z}}$$

where

$$\frac{\partial H}{\partial \bar{z}} : \mathbb{C} \to \mathbb{C}$$

which

$$\frac{\partial}{\partial \bar{z}}\frac{\partial H}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial p_i} + i\frac{\partial}{\partial q_i}\right)\frac{1}{2}\left(\frac{\partial H}{\partial p_i} + i\frac{\partial H}{\partial q_i}\right) = \frac{1}{4}$$

- Canonical quantization will give Schrödinger-Heisenberg representation

---

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
      - Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form

And it has 10 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
      - Constant of a flow in $\mathbb{R}^n$
      - Euler's iterative solution method for first order ODEs
      - Existence of integral curves of vector fields in $\mathbb{R}^n$
      - Fixed points
      - Flow of a vector field in $\mathbb{R}^n$
      - Graph $\to$ polynomial ODE
      - Gradient flows on $\mathbb{R}^n$
      - Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form
      - Probability distribution of flow
      - Volume change by flows on $\mathbb{R}^n$