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Derivative of maps $U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$

Intuition

Let

$$f : U \text{ (open)} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p \mathbb{R}^n \rightarrow T_p \mathbb{R}^m$$

such that

$$f(p + v) \approx f(p) + \mathfrak{D}_p f(v)$$

Definition. Derivative of a map $U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$ at a point

Let

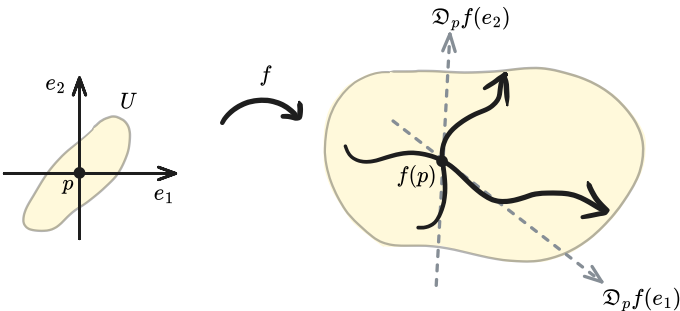
$$f : U \text{ (open)} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p \mathbb{R}^n \rightarrow T_p \mathbb{R}^m$$

such that

$$\begin{aligned} \lim_{v \rightarrow 0} \frac{f(p + v) - f(p) - \mathfrak{D}_p f(v)}{|v|} &= 0 \\ \iff \lim_{v \rightarrow 0} \frac{|f(p + v) - f(p) - \mathfrak{D}_p f(v)|}{|v|} &= 0 \end{aligned}$$



$$\begin{aligned} \iff \lim_{x \rightarrow p} \frac{|f(x) - f(p) - \mathfrak{D}_p f(x - p)|}{|x - p|} &= 0 \\ \iff \forall \epsilon > 0 \exists r(\epsilon) > 0 : \\ \forall x \in B_{r(\epsilon)}(p), |f(x) - f(p) - \mathfrak{D}_p f(x - p)| &\leq \epsilon |x - p| \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

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And it has 10 siblings.

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