ⓘ Info

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open and closed sets in a metric space

open sets

Definition. Open set

A set $S \subseteq M$ is an **open set** if all of its points are interior points.

> **Definition.** Interior point
>
> Let $S \subseteq M$ and $x \in S$. Then x is an interior point of S if
>
> $$\exists r > 0;\ r\text{–}B_M(x) \subseteq S.$$

- M is open.
- $\emptyset$ is open.

Definition. Interior of S

$$\operatorname{int}(S) := \text{interior points of } S$$

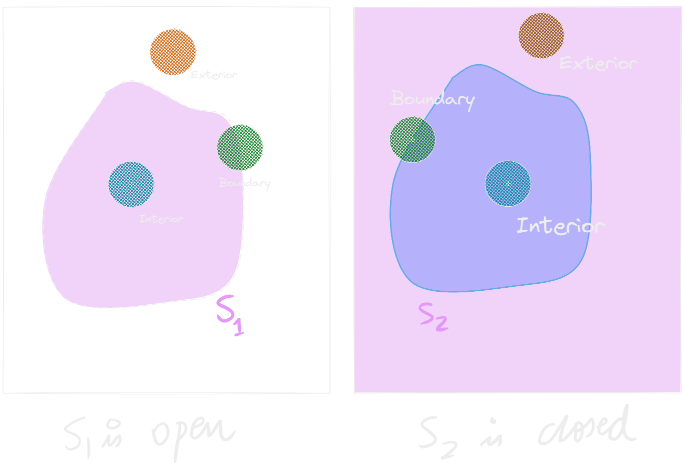
S is open $\iff$ $S = \operatorname{int}(S)$ $\iff$ $S \subseteq \operatorname{int}(S)$

Every $r\text{–}B_M$ is open.

closed sets

Definition. Closed set

A set $S \subseteq M$ is an **closed set** if $M \setminus S$ is an open set.

- 

$$S \text{ is closed} \iff \partial S \subseteq S$$

limit points and closed sets

A subset S of a metric space M is closed iff it contains all its accumulation points/limit points, i.e.

$$S \text{ is closed} \iff \operatorname{accu}(S) \subseteq S$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - open and closed sets in a metric space

And it has 9 siblings.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - adherent, accumulation points, closure of a subset in a metric space
      - Balls in a metric space
      - Totally bounded subsets of a metric space
      - Compact subsets of a metric space
      - Covering of a subset of a metric space
      - Open cover of a compact metric space
      - Function between metric spaces
      - interior, exterior, boundary points in a metric space
      - open and closed sets in a metric space