Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on March 29, 2024 1:32:43 AM,
was last modified on May 17, 2026 7:36:52 PM, and
was exported on September 7, 2026 3:31:14 PM.

Iterations of a smooth map on a smooth manifold

Definition. Hyperbolic fixed points of a smooth manifold diffeomorphism

A *fixed point* p of a diffeomorphism $f : M \to M$ of a smooth manifold M is said to be **hyperbolic** if all (including complex) eigenvalues of $\mathrm{d}_p f : T_p M \to T_p M$ have absolute value different from 1.

For such a fixed point p, the tangent space splits into $\mathrm{d}_p f$-invariant subspaces called **stable** and **unstable** tangent subspaces

$$T_p M = E_p^{\mathsf{u}} \oplus E_p^{\mathsf{s}}$$

where $E_p^{\mathsf{u}}, E_p^{\mathsf{s}}$ are generalized subspaces with (complex) eigenvalues with absolute value more and less than 1 respectively.

Moreover, for such a fixed point p and its neighborhood U we have the **local stable and unstable subsets**

$$W^{\mathsf{s}}(p, U) = \{x \in U | \lim_{n\to\infty} f^n(x) = p\}$$
$$W^{\mathsf{u}}(p, U) = \{x \in U | \lim_{n\to\infty} f^{-n}(x) = p\}$$

The global stable and unstable subsets are $W^{\mathsf{s}}(p, M)$ and $W^{\mathsf{u}}(p, M)$.

(Discrete Hartman-Grobman theorem) If p is a *hyperbolic fixed point* of a diffeomorphism $f : M \to M$ on a smooth manfold M then f near p is **topologically conjugate** to $\mathrm{d}_p f : T_p M \to T_p M$ near $0 \in T_p M$.

This allows us to classify hyperbolic fixed points locally by classifying the same only for $\mathbb{R}$-linear maps.

Let $f : M \to M$ is a $\mathcal{C}^k$-diffeomorphism of a $\mathcal{C}^k$-manifold M and p is a hyperbolic fixed point of f. Then there exists a neignbourhood U of p such that the local *stable* and *unstable* subsets $W^{\mathsf{s}}(p, U)$ and $W^{\mathsf{u}}(p, U)$ are $\mathcal{C}^k$-embedded submanifolds of M with

$$T_p W^{\mathsf{s}}(p, U) = E_p^{\mathsf{s}},\ T_p W^{\mathsf{u}}(p, U) = E_p^{\mathsf{u}}$$

The global stable and unstable sets are manifolds, but *immersed* not embedded submanifolds in general.

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And it has 5 siblings.

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