Riemannian connection

- *Torsion-free*

$$\nabla_X Y - \nabla_Y X = [X, Y]$$

- *metric-compactible*

$$X(g(Y,Z)) = g(\nabla_X, Z) + g(Y, \nabla_X Z)$$

$$\begin{aligned} \nabla : \mathrm{Vec}(M) &\to \mathrm{Vec}(M) \otimes \Omega^1(M) = ?\mathbf{\Gamma}(T^*M \otimes TM) \\ X &\mapsto \nabla_{(-)} X \end{aligned}$$

$$[X, Y] \mapsto \nabla[X, Y] = ?$$

expression

We compute

$$X(g(Y,Z)) + Y(g(X,Z)) - Z(g(X,Y))$$

and obtain

$$g(\nabla_X Y, Z) = \begin{matrix} X(g(Y,Z)) + Y(g(Z,X)) - Z(g(X,Y)) \\ -g(Y,[X,Z]) - g(Z,[Y,X]) + g(X,[Z,Y]) \end{matrix}$$

the covector field of $\nabla_X Y$ is

$$X(g(Y,-)) + Y(g(-,X)) - \mathrm{d}(g(X,Y)) - g(Y,[X,-]) - g(-,[Y,X]) + g(X,[-,Y])$$

in a chart

On a chart (U, x) we have

$$\nabla_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j} = \sum_m \Gamma_{ij}^m \frac{\partial}{\partial x_m}$$

because $\left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right] = 0$ we have

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Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Riemannian connection

And it has 18 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Universal cover of semi-Riemannian manifolds
      - Curvature of a Riemannian manifold
      - Einstein manifolds
      - Exponential map from tangent bundle
      - Geodesics past conjugate points in a Riemannian manifold
      - Riemannian manifolds with ergodic geodesic flow
      - Isometries of a semi-Riemannian manifold
      - Laplacian on a Riemannian manifold
      - Parallel transport on a curve in a semi-Riemannian manifold
      - Conjugate points in a semi-Riemannian manifold
      - Quotients of Riemannian manifolds
      - Complete Riemannian manifolds
      - Riemannian connection
      - Riemannian submanifolds
      - Totally geodesic submanifolds of semi-Riemannian manifolds
      - Tangent bundle of a Riemannian manifold
      - Unit tangent bundle of a Riemannian manifold
      - Vector fields on Riemannian manifolds