ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on August 20, 2026 6:51:45 PM,
was last modified on August 28, 2026 3:12:26 PM, and
was exported on September 7, 2026 3:25:05 PM.

Quasi-geodesics in a metric space

Definition. Arcs, rays and lines

A path

$$\gamma : I(\text{interval}) \subseteq \mathbb{R} \to M$$

is called	if
arc/segment joining $p, q \in M$	$I = [a, b]$ and $\gamma(0) = p, \gamma(1) = q$
ray originating at $p \in M$	$I = [0, \infty)$ and $\gamma(0) = p$
line	$I = \mathbb{R}$

Definition. Quasi-isometries between metric spaces

Let (X, d_X) and (Y, d_Y) be metric spaces. A map

$$f : X \to Y$$

- is a (c, b)-**quasi-isometric embedding** if there are constants $c, b > 0$ such that $\forall x_1, x_2 \in X$

$$\frac{1}{c} d_X(x_1, x_2) - b \leq d_Y(f(x_1), f(x_2)) \leq c d_X(x_1, x_2) + b$$

- is **finite distance** from $f' : X \to Y$ if

$$\exists c > 0 : \forall x \in X, d_Y(f(x), f'(x)) \leq c$$

- has a **quasi-inverse** $g : Y \to X$ if $g \circ f$ has finite distance from Id_X and $f \circ g$ has finite distance from Id_Y
- is a **quasi-isometry** if it is a *quasi-isometric embedding* with a *quasi-inverse* $g : Y \to X$ which is also a quasi-isometry

Definition. Quasi-geodesics in a metric space

Let (M, d) be a metric space. A path

$$\gamma : I(\text{interval}) \subseteq \mathbb{R} \to M$$

is called a **quasi-geodesic** if it is a quasi-isometric embedding.

Therefore, we have

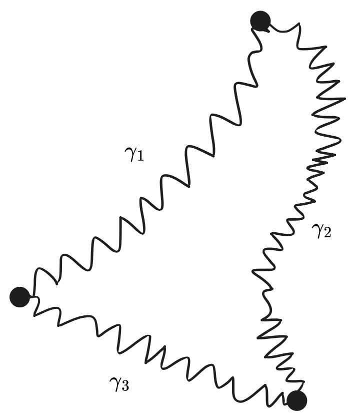
- *quasi-geodesic segments* which are quasi-isometric embeddings $[0, L] \to M$
- *quasi-geodesic rays* which are quasi-isometric embeddings $[0, \infty) \to M$
- *quasi-geodesic lines* which are quasi-isometric embeddings $\mathbb{R} \to M$

Definition. Quasi-geodesic triangles

Let (X, d) be a metric space. A **quasi-geodesic triangle** is a triple

$$\gamma_1, \gamma_2, \gamma_3 : [0, L_1], [0, L_2], [0, L_3] \to X$$

of *quasi-geodesic segments*



such that

$$\gamma_1(L_1) = \gamma_2(0)$$
$$\gamma_2(L_2) = \gamma_3(0)$$
$$\gamma_3(L_3) = \gamma_1(0)$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - quasi
      - Quasi-geodesics in a metric space

And it has 2 siblings.

- structures based on sets
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