Info

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Lamplighter group on $\mathbb{Z}_2$

Definition. The Lamplighter group L_2

Consider the left shift on the group $\bigoplus_{\mathbb{Z}} \mathbb{Z}_2$

$$\sigma(\ldots, x_0, x_1, \ldots) = (\ldots, x_1, x_2, \ldots)$$

which generates

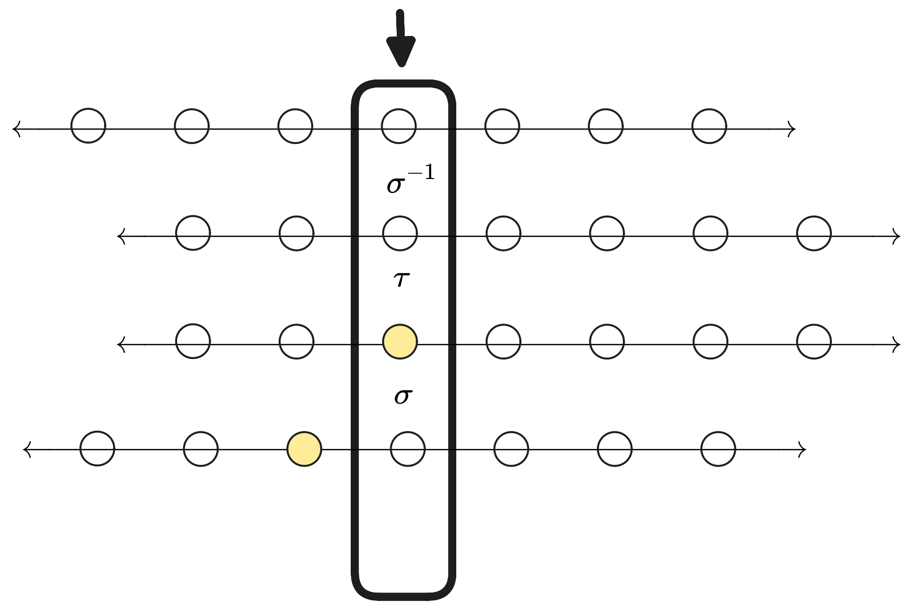
$$\mathbb{Z} \to \mathsf{AutGrp}\left(\bigoplus_{\mathbb{Z}} \mathbb{Z}_2\right)$$

$$\underline{x} \overset{n}{\mapsto} \sigma^n(\underline{x})(i) = x_{i+n}$$

The group $L_2 := \mathbb{Z} \ltimes_\sigma \bigoplus_{\mathbb{Z}} \mathbb{Z}_2$ where the multiplication is

$$(n, \underline{x})(m, \underline{y}) = (n + m, x + \sigma^n(\underline{y}))$$

The Lamplighter group L_2 acts on the **space of all lamp stands** $\bigoplus_{\mathbb{Z}} \mathbb{Z}_2$.



- The group is generated by

$$(1, \underline{0}), (0, e_0)$$

- The group is solvable as

$$[L_2, L_2] = \bigoplus_{\mathbb{Z}} \mathbb{Z}_2$$

$$\left\{ \begin{bmatrix} t^k & p \\ 0 & 1 \end{bmatrix} \in GL(2)(\mathbb{Z}_2[t, t^{-1}]) \middle| p \in \mathbb{Z}[t, t^{-1}], k \in \mathbb{Z} \right\}$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - Lamplighter group on $\mathbb{Z}_2$

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - B^n
  - $\mathrm{BS}(1,2)$
  - $(\mathbb{C}, +, \times)$
  - $D^* := D \setminus \{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
  - H_2^3
  - $[a, b]$
  - Figure 8 knot
  - Lamplighter group on $\mathbb{Z}_2$
  - $\beta\mathbb{N}$
  - $\#_2 \mathbb{R}P^2$
  - $\prod_{i \in \mathbb{N}} \{1, \ldots, n_i\}$
  - $\mathbb{R}P^n$
  - $(\mathbb{Q}, +, \times, <)$
  - $\mathbb{Q}_{\hat{p}}$
  - $\overline{\mathbb{Q}}$
  - $\mathbb{Q}_{\text{constr}}$
  - $\mathbb{Q}(\xi_n)$
  - $\mathcal{O}_{\overline{\mathbb{Q}}}$
  - $(\mathbb{R}, +, \times, <, \sup)$
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset

- $\mathbb{R}^2 \times S^1$
- $\mathbb{R}^n$
- $\mathbb{R} \times S^1$
- $\mathbb{R} \times S^2$
- S^1
- $S^1 \wedge S^1$
- $S^1 \times S^n$
- S^2 and $\mathbb{C}\boldsymbol{P}^1$
- Mapping torus of the antipodal map of S^2
- S^3
- S^n
- I-bundle
- Symmetric group
- T
- T^2
- T_g^2
- $T_{0,0,3}^2$
- $\mathbb{R}^2 \setminus \{0\}$
- Three $T_{1,0,1}^2$ glued at their boundaries
- $T_{1,1,0}^2$
- T_2^2
- $T_g^2 \times [0,1]$
- $\#_{\mathbb{N}} T^2$
- Thompson's group
- T^n
- 3-valent tree
- 4-valent tree
- $[0,1]^{\mathbb{N}}$ with product topology
- Whitehead manifolds
- $\mathbb{Z}$
- $\mathbb{Z}[\sqrt{m}]$
- $\mathbb{Z}[X]$
- $\mathbb{Z}^{\mathbb{N}}$
- $\mathbb{Z}[i]$
- $\mathbb{Z}[i\sqrt{5}]$