Info

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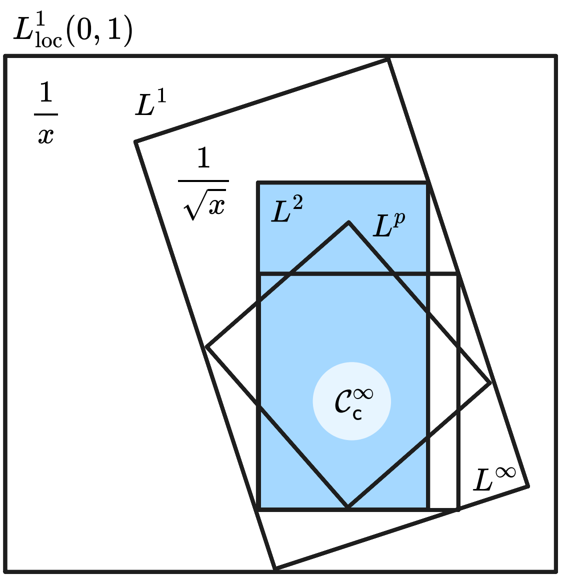
Measurable functions $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$

We have

$$\begin{array}{ccccc} \begin{array}{c} L^1(0,1) \\ = L^1[0,1] \\ = L^1_{\mathsf{loc}}[0,1] \end{array} & > & \begin{array}{c} L^2(0,1) \\ = L^2[0,1] \end{array} & > & \cdots \\ \wedge & & \wedge ? & & \\ L^1_{\mathsf{loc}}(0,1) & > & L^2_{\mathsf{loc}}(0,1) & > & \cdots \end{array}$$

[1]



Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$

And it has 15 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Functions with uniformly bounded mean oscillations on cubes
      - Decomposition of $f \in L^p$ into bounded and unbounded parts
      - Calderon-Zygmund decomposition
      - Lebesgue density of measurable sets
      - Functions with uniformly bounded mean oscillations on dyadic cubes
      - Volterra operator $\int_{[0,-]} : L^1_{\mathsf{loc}}[0,1) \to \mathcal{C}[0,1)$
      - Measurable functions on $\mathbb{R}^n$
      - (ϵ, n)-measurable function
      - Integrability and integral of measurable functions on $\mathbb{R}^n$
      - Hardy-Littlewood maximal functions of $L^1_{\mathsf{loc}}(\mathbb{R}^n)$
      - Lebesgue averaging and differentiation
      - Integrals of a monotonically converging sequence of functions
      - Lebesgue integral from measure of undergraph
      - $L^{p,q}$
      - $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$

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1. **Proposition:** Let (X, μ) be a measure space such that $\mu(X) < \infty$. We have

$$0 < p < q \leq \infty \implies L^p(X,\mu) \leq L^q(X,\mu)$$
$$\text{and } \|f\|_p \leq \|f\|_q \, \mu(X)^{\frac{1}{p}-\frac{1}{q}}$$

Thus, in particular

- for $\mu(X) = 1$

$$L^1 \geq L^2 \geq \cdots \geq L^\infty$$
$$\| \ \|_1 \leq \| \ \|_2 \leq \cdots \leq \| \ \|_\infty$$

- the inclusions

$$\iota : (L^p(X,\mu), \| \ \|_p) \hookrightarrow (L^q(X,\mu), \| \ \|_q)$$

are continuous(?) for $0 < p < q < \infty$ and has a dense image.

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